MONTHLY NOTICES

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 98 No. 1

MEETING OF 1937 NOVEMBER 12

Dr. H. Spencer Jones, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

William Bradshaw Housman, The Observatory, Seaton, Workington, Cumberland ;

Frederick George Jones, 8 Clifton Vale, Clifton, Bristol ; and

James Barrett Livingston, Warwick Close, Deansway, Bishop's Avenue, London, N. 2.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Robert d'Escourt Atkinson, Royal Observatory, Greenwich, London, S.E. 10 (proposed by H. Spencer Jones) ;

Edwin A. Baker, D.Sc., Royal Observatory, Edinburgh (proposed by R. A. Sampson) ;

William Henry Barton, jun., Associate-Curator of Astronomy and of the Hayden Planetarium, American Museum of Natural History, New York City, U.S.A. (proposed by Clyde Fisher) ;

John Ralph Bazin, 84 Hendon Lane, London, N. 3 (proposed by F. J. Sellers) ;

Miss Cicely Mary Botley, F.R.Met.Soc., 17 Holmesdale Gardens, Hastings (proposed by F. J. W. Whipple) ;

Hermann A. Brück, Solar Physics Observatory, Cambridge (proposed by F. J. M. Stratton) ;

George Arthur Bull, B.Sc., F.R.Met.Soc., Meteorological Office, Royal Air Force, Felixstowe, Suffolk (proposed by G. S. LeBeau) ;

Rev. Reginald Henry Consterdine, Lisadel, Wilmslow, Manchester (proposed by W. H. Steavenson) ;

E. T. Holdaway, Government Astronomer for Time Keeping, Queensland Observatory, Brisbane, Australia (proposed by A. K. Chapman) ;

Basil Munroe Holden, B.A., 15 Hampstead Lane, Highgate, London, N. 6 (proposed by W. M. Smart) ;

Clifford George James, Fairey Aviation Co., Ltd., Hamble (Seaplane Section), Hants, and 5 Longmore Avenue, Woolston, Southampton, Hants (proposed by Peter Doig) ;

Eric Mervyn Lindsay, M.A., M.Sc., Ph.D., Director of the Observatory, Armagh, Ireland (proposed by Harlow Shapley) ;

Miss Flora M. McBain, H.M. Nautical Office, Royal Naval College, Greenwich, London, S.E. 10 (proposed by J. A. Carroll) ;

Alexander Menzies, Royal Observatory, Cape of Good Hope, South Africa (proposed by J. Jackson) ;

Rev. W. O'Leary, S.J., Director of the Riverview Observatory, Sydney, N.S.W., Australia (proposed by James Nangle) ;

Gordon Edmond Patston, 55 Woodbourne Avenue, Streatham, London, S.W. 16 (proposed by W. H. Steavenson) ;

John Guy Porter, B.Sc., F.C.S., 13 Cleave Road, Gillingham, Kent (proposed by M. Davidson) ;

George Ormerod Rawstron, 216 Pullman Court, Streatham Hill, London, S.W. (proposed by Kennet Pollock) ;

Reginald Campbell Smith, 99 Broomhill Drive, Glasgow, W. 1 (proposed by T. L. MacDonald) ; and

Joseph Bernard Godsell Turner, Royal Observatory, Cape of Good Hope, South Africa (proposed by J. Jackson).

Two hundred and thirty presents were announced as having been received since the last meeting, including :—

Sir Frank Dyson and R. v. d. R. Woolley, Eclipses of the Sun and Moon ;

Harold Jeffreys, The Problem of Oceanic Structure ;

The Physical Society, Report of a Joint Committee of the Chemical Society, The Faraday Society and the Physical Society on Symbols for Thermodynamical and Physico-Chemical Quantities and Conventions relating to their Use ;

G. C. McVittie, Cosmological Theory.

The President announced the death, at the age of ninety-seven, of Mr. Richard Inwards, who was elected a Fellow of the Society in 1861, and was the oldest Fellow of the Society both in date of election and in actual age.

The President further announced that the rooms of the Society would be open on Wednesday evenings until nine o'clock throughout the session.

THE CONTOURS OF LINES AT THE EXTREME LIMB OF THE SUN.

R. v. d. R. Woolley.

1. The Cambridge Eclipse Expedition in 1936 brought back from Kamishari spectra of the extreme limb of the Sun just before totality, from which Thackeray has produced some very remarkable contours showing the change over from absorption lines on the limb itself to emission lines in the chromosphere (fig. 1). These very interesting contours invite a development of the theory of the formation of lines at the extreme limb. A little consideration has convinced the present writer that the topic is of very great importance to the theory in general, as the conditions are very critical and will show up weaknesses in a mathematical representation which will do well enough when it is only used to predict contours for the middle of the solar disc, or for a star as a whole. Cloud interfered with the Kamishari exposures, and the observational material is accordingly incomplete, so that it is clearly desirable that the observations should be repeated on another occasion; yet a theoretical discussion at the present time will not be premature if it focusses attention on the importance of the subject and perhaps opens up the possibility of obtaining observations in such a manner as to afford a direct comparison with theory at special points.

Soon after Unsöld produced his observations of line contours and compared them with theory, the theory itself was taken forward from the point at which Schuster had left it by Milne and Eddington. The theory of line formation was one of strictly monochromatic radiative equilibrium saving only the effect of superelastic and inelastic collisions, that is to say that it was assumed that the scattering was coherent and that there was no interlocking between spectral lines, assumptions which Eddington criticised severely. This theory still stands as the only completely worked-out theory of line absorption—it has been very completely worked out by Pannekoek. When compared with observation it met with two difficulties, the first of which was quickly disposed of to the satisfaction of many writers on the subject. This was the fact that relative intensities of lines within a multiplet did not appear at first to agree with theoretical predictions. However, the theory only concerns itself with the transfer of radiation through the atmosphere with a given absorption coefficient, nothing being said about the dependence of the absorption coefficient on the frequency, and the multiplet intensities can be set right by adopting a suitable distribution of the absorption coefficient with frequency, since quantum theory is not sufficient completely to determine the distribution in practical cases.

The second difficulty was raised by the central intensities of the lines. The theory demands a central intensity less than 1 per cent. of that of the continuous background, and early observations indicated a much higher

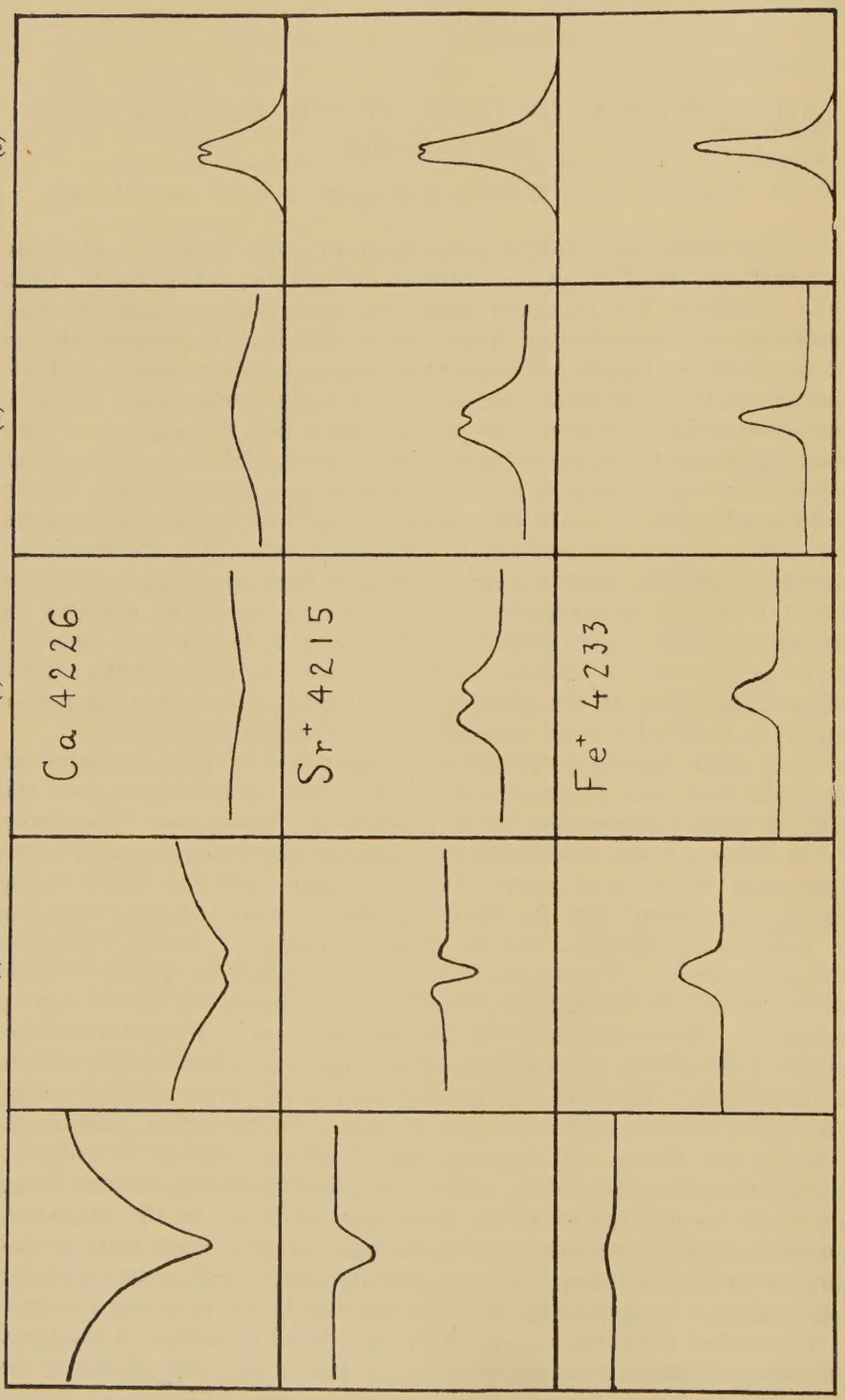

FIG. 1.—*Thackeray's observational contours showing the transition from emission to absorption. The intensity of the continuous spectrum diminishes by half from (a) to (d); (e) corresponds to a point just outside the limb.*

central intensity. Later observations have confirmed the high central intensities in the case of hydrogen lines, but show central intensities of metallic lines which are not inconsistent with the monochromatic theory of line formation.

We are accordingly without a simple observational test of the validity of the simple theory. It was shown by H. H. Plaskett that variation in line contour from the centre of the disc to the limb may be expected to throw some light on the process of line formation, but unfortunately the variation except at the extreme limb is rather small, and nothing completely decisive emerges unless the observations go right to the limb. It appears that eclipse observations of the spectrum of the extreme limb afford the most critical test of any theory of the formation of absorption lines.

2. The theory now developed is that stated explicitly by Eddington,* in which it is assumed that the radiative transfer is strictly monochromatic saving only the effect of collisions. Eddington's equation in his own notation is

$$\cos\theta\frac{d\mathcal{J}_\nu'(\theta)}{d\tau}=(1+\eta)\mathcal{J}_\nu'(\theta)-(1-\epsilon)\eta\mathcal{J}_\nu'-(1+\epsilon\eta)\mathcal{J}_T. \qquad (1)$$

If we assume that $\mathcal{J}_T=H(2+3\tau)$ and write $q^2=3(1+\eta)(1+\epsilon\eta)$, the solution for η constant with respect to τ is

$$\mathcal{J}_\nu'=\mathcal{J}_T+Ae^{-q\tau}, \qquad (2)$$

$\mathcal{J}_\nu'$ being the radiation intensity averaged in all directions θ. The constant A is given by

$$A=\frac{-2H\eta}{1+\eta+\frac{2}{3}q}.$$

Following H. H. Plaskett,† we assume that this (approximate) solution for $\mathcal{J}_\nu'$ is correct and solve equation (1) for $\mathcal{J}_\nu'(\theta)$. The solution for the contour of the line is

$$r(\theta)=\frac{2}{2+3\cos\theta}\left\{1+\frac{3}{2}\frac{\cos\theta}{1+\eta}-\frac{(1-\epsilon)\eta^2}{(1+\eta+\frac{2}{3}q)(1+\eta+q\cos\theta)}\right\}. \qquad (3)$$

Since the pressure in the chromosphere and reversing layer is so low that the value of ϵ due to collisions is very low, the presence of the term ϵ if demanded by comparison of the contours with observation must be regarded as a failure of the monochromatic theory in the sense of § 1. The use of ϵ for this purpose was first suggested by Strömgren.‡

3. The foregoing equations (2) and (3) apply in the case where the material is stratified in planes, and the conditions at the limb suggest that the curvature of the layers in the Sun's atmosphere may influence the result. It is therefore necessary to show that *curvature produces a negligible effect on equation* (2). The spherical case has been investigated by

* *M.N.*, **89**, 620, 1929.
† *M.N.*, **91**, 870, 1931.
‡ *Zs. f. Astrophys.*, **10**, 237, 1935.

Chandrasekhar. His equation (86) gives the following differential equation * for the flow of radiation:—

$$\frac{1}{r^2}\frac{d}{d\tau}\left\{r^2\frac{d(\mathcal{J}_\nu' - \mathcal{J}_T)}{d\tau}\right\} = q^2(\mathcal{J}_\nu' - \mathcal{J}_T). \qquad (1')$$

Chandrasekhar solves this equation for a case which is quite unsuitable for representing the Sun. It is necessary to specify τ as a function of r (a step which is unnecessary in the plane case). Chandrasekhar chooses $\tau \propto r^{-n}$, which has no finite boundary. It seems better to suppose $\tau = a \log (r/R)$, where R is the radius of the Sun and a is negative. (It must be noted that only values of $r/R > 0{\cdot}9$ are at all relevant.) The solution of (1′) is then

$$\mathcal{J}_\nu' - \mathcal{J}_T = Ke^{\lambda\tau}, \qquad (2')$$

where $\lambda = -\frac{1}{a} \pm \sqrt{\frac{1}{a^2} + q^2} = \pm q - \frac{1}{a}$ if a is large. Now the whole extent of the reversing layer is of order 150 km. at a generous estimate, so that a is at least of order 10^4. Then $\lambda = \pm q$ to a high degree of approximation and (2′) is identical with (2), showing that the curvature does not affect (2). This result is physically obvious.

The introduction of the relation $\tau = a \log (r/R)$ having served its purpose in this connection, we assume that other distributions will behave in much the same way as regards difference between the spherical and plane cases. In the remainder of the paper we deal only with the plane case (as regards equation (2) or (2′)), and it is then unnecessary to specify τ as a function of r.

4. The variation of the contour (3) with varying θ is not sufficient to account for the phenomena at the extreme limb. Interest centres on the case where the line of sight passes very close to the edge of the disc, so that the optical thickness along the line of sight is not very great.

Let P (fig. 2) be a point on the line of sight, and let Q be the intersection of the line of sight with the near side of the solar surface (supposed to have a definite boundary). Produce the radius vector from the centre of the Sun through P to cut the surface at R. Let $PQ = \xi$ and write $\tau(\xi) = \int k\rho d\xi$, the symbol τ without qualification referring to the optical depth PR. Let

$$QS = \xi_1, \qquad \int_0^{\xi_1} k\rho d\xi = \tau_1.$$

Then the efflux at Q into solid angle $\delta\omega$ is

$$E = \frac{\delta\omega}{4\pi}\int_0^{\xi_1}\{(1-\epsilon)\eta\mathcal{J}_\nu'(P) + (1+\epsilon\eta)\mathcal{J}_T\}\exp\{-(1+\eta)\tau(\xi)\}k\rho d\xi$$

$$= \frac{\delta\omega}{4\pi}\int_0^{\xi_1}\{(1+\eta)H(2+3\tau) + (1-\epsilon)\eta Ae^{-q\tau}\}\exp\{-\overline{1+\eta}\tau(\xi)\}d\tau(\xi).$$

But τ along the radius vector is always negligible. Outside the line the efflux is $2H(1-e^{-\tau_1})$, and where the disc has begun to fade out τ_1 will be

* *M.N.*, 94, 455, 1934.

less than, say, 10, while τ along the radius vector is of order $\tau_1 \times \xi_1/$(the radius of the Sun), and may be neglected. The efflux is therefore

$$E = \frac{\delta\omega}{4\pi}\left\{2H + \frac{(1-\epsilon)\eta A}{1+\eta}\right\}\{1 - \exp -(1+\eta)\tau_1\}. \qquad (4)$$

When τ_1 is large (4) is identical with (3), in which one has set $\cos\theta = 0$. Fig. 3 (*a*) and (*b*) illustrates this equation. There are two series of contours, one for $\epsilon = 0$ and one for $\epsilon = 0{\cdot}2$, each series being drawn for $\tau_1 = \infty$, 0·4, 0·1 and 0·01. The spacing of η with $(\nu - \nu_0)$ is immaterial to the present argument and I have set $\eta \propto (\nu - \nu_0)^{-2}$ for simplicity.

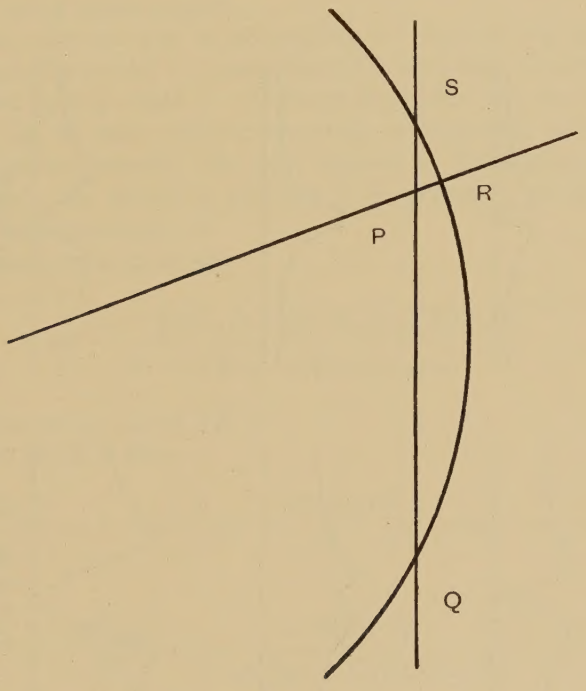

FIG. 2.

When $\epsilon = 0$ the centre of the line is black at the centre of the disc, and the line must necessarily be reversed at the limb. The curve for $\epsilon = 0$ shows a rough resemblance to Thackeray's contour for *Ca* 4226 A. It is true that the theoretical curves show zero central intensities which are not shown by the observations, but it is possible that these are in fact low central intensities which would be shown by an instrument of higher resolving power. The observed emission (*e*) is much too large for equation (4). Again, when $\epsilon = 0{\cdot}2$ the contour at the limb is not reversed and there is some resemblance to Thackeray's contour for Sr^+ 4215 A. But it must be noted that $\epsilon = 0{\cdot}2$ requires a high central intensity for Sr^+ 4215 A. on the disc; while there is no reliable observation of this quantity, the very similar line Sr^+ 4078 A. has a central intensity of only 2 per cent. of the continuous

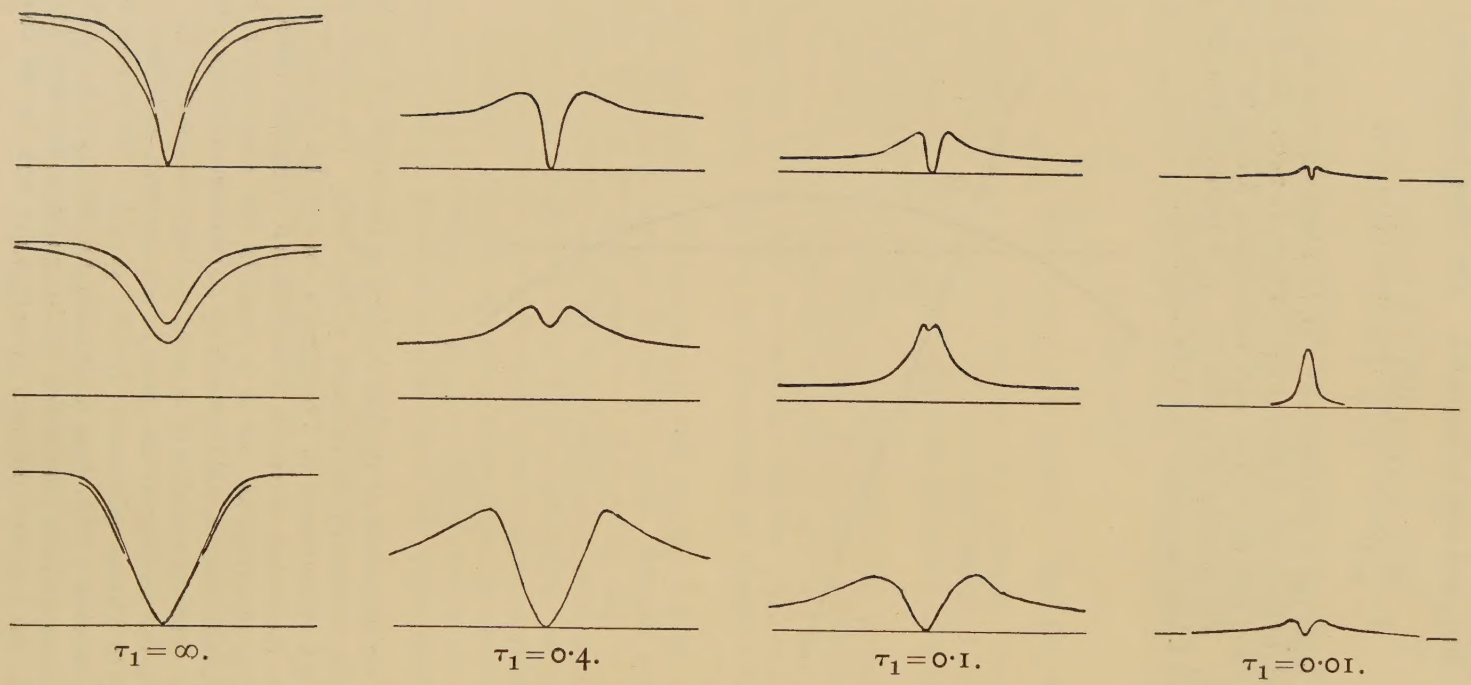

FIG. 3.—*Contours computed from equations* (4) *and* (5). (a) η *constant*, $\epsilon = 0$. (b) η *constant*, $\epsilon = 0{\cdot}2$. (c) η *decreasing with* τ, $\epsilon = 0$. *In each case the left-hand figure shows the mean contour for the disc as well as the contour at the limb.*

background according to Redman, and the high value of ϵ cannot be tolerated.

The theory as developed in this paragraph fails to account for the Kamishari results, but we must notice the quite natural appearance of shoulders of the line. These occur when τ_1 is moderate, because when the disc is semi-transparent to radiation outside the line it is still opaque to radiation in the line frequencies, which hold up their values of r while the continuous spectrum is fading out with decreasing τ_1. This effect gives the appearance of a chromosphere in the sense of an emission line projecting above the limb, as seen in the continuous spectrum even in the case η constant, in which general absorption holds right up to the boundary beyond which no selectively scattering atoms project.

5. The most obvious way of attempting to improve the treatment of § 4 is to abandon the model η constant and to introduce a model in which η varies with the optical depth τ. There is at present only one case known by which this can be done without resorting to numerical solution of the differential equations, namely, the case treated by Eddington,* in which $(1+\eta)^{-1} = \alpha + \beta\tau$. To obtain a solution it is necessary to suppose that $\epsilon = 0$ or that $1 + \epsilon\eta = \text{constant} = C$.

The mean contour is given by

$$r = \frac{\alpha - \sqrt{\tfrac{1}{3}\alpha C}\,.\,(2+\beta/C)K_0'(x_0)/K_0(x_0)}{1 - 2\sqrt{\tfrac{1}{3}\alpha C}\,.\,K_0'(x_0)/K_0(x_0)},$$

where the argument $x_0 = (12\alpha C)^{\frac{1}{2}}\beta^{-1}$.

The solution for E is then

$$E = \frac{\delta\omega}{4\pi} H \left\{ 2 + (1 - \alpha C)\left(\beta/C + \frac{AK_0(x_0)}{H} \right) \right\} \{1 - \exp(-\alpha^{-1}\tau_1)\} \qquad (5)$$

together with

$$-\frac{AK_0(x_0)}{H} = \frac{2 + \beta/C - 2\alpha}{1 - 2\sqrt{\tfrac{1}{3}\alpha C}\,.\,K_0'(x_0)/K_0(x_0)}. \qquad (6)$$

The case $\epsilon = 0$ is a special case of (5) and (6) in which $C = 1$.

Numerical work on these equations shows that nothing is gained substantially over the case η constant. The shoulders of the reversal are intensified if η is made to increase towards the surface, but as in the case η constant a chromospheric emission is only obtained by giving a high value to ϵ and to the central intensity of the line on the disc. One case is illustrated in fig. 3 (*c*).

6. A more definite representation of the chromosphere as a layer physically distinct from the reversing layer may be obtained by dividing our mathematical representation of the solar atmosphere into two parts. Above a certain dividing surface the continuous absorption coefficient k is put zero, and below this boundary we have η constant with τ or $(1+\eta)^{-1} = \alpha + \beta\tau$.

* *M.N.*, **89**, 630, 1929.

In the simplest case of η constant and no attempt to introduce an ϵ term into the upper atmosphere, we have above the boundary

$$\left.\begin{aligned} \mathcal{J}_\nu' &= H_\nu'(2+3\sigma_\nu) \\ H_\nu' &= \text{constant} \end{aligned}\right\}, \tag{7}$$

where $\sigma_\nu = -\int l_\nu \rho dx$. The boundary is set at $\sigma = \sigma_1$, and below the boundary

$$\left.\begin{aligned} \mathcal{J}_\nu' &= \mathcal{J}_T + Ae^{-q\tau} \\ 3(1+\eta)H_\nu' &= 3H - qA \end{aligned}\right\}. \tag{8}$$

The boundary conditions are the continuity of $\mathcal{J}_\nu'$ and H_ν' at $\tau = 0$, $\sigma = \sigma_1$. They give for A and H_ν'/H

$$\left.\begin{aligned} A\{3(1+\eta)+q(2+3\sigma_1)\} &= H\{9\sigma_1 - 6\eta\} \\ \frac{H_\nu'}{H} &= \frac{1+\frac{2}{3}q}{1+\eta+\frac{2}{3}q(1+\frac{3}{2}\sigma_1)} \end{aligned}\right\}. \tag{9}$$

The equations (7), (8) and (9) specify the quantity $\mathcal{J}_\nu'$ as we proceed inwards along the radius vector, and by a suitable manipulation of the algebra it is easy to integrate the efflux along the line of sight so as to produce an expression analogous to (4) or (5).

7. In this section the emission from the chromosphere in the case where the line of sight passes outside the limb is evaluated. Let P (fig. 4) be a point on the line of sight, and let OC be the normal to the line of sight passing through the Sun's centre. Let $PC = \xi$, counted positive towards the observer. Let n_P be the number of atoms per c.c. at P, and let N_P be the number of atoms in a column of unit cross-section between the observer and P. Let s_ν be the atomic scattering coefficient so that $ns_\nu = \rho l_\nu$.

Then the efflux into a solid angle $\delta\omega$ along the line of sight is

$$E = \frac{\delta\omega}{4\pi}\int_{-\infty}^{\infty} n_P s_\nu (\mathcal{J}_\nu')_P \exp(-N_P s_\nu) d\xi. \tag{10}$$

The chromospheric atoms may be supposed to be distributed according to an exponential law, so that if h is the height measured radially,

$$n_h = n_0 \exp(-2\lambda^2 R h),$$

where R is the radius of the Sun. Then since ξ/R is small for all effective contributions to the integral (10),

$$n_P = n_c \exp(-\lambda^2\xi^2).$$

Also

$$N_P = \frac{n_c\sqrt{\pi}}{2\lambda}[1 - \Phi(\lambda\xi)]$$

(notice a result required in § 10, that the total number of atoms along the line of sight $N = \frac{n_c\sqrt{\pi}}{\lambda}$, i.e. $N \propto n_c$), where

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2} dx.$$

The value of $(\mathcal{J}_\nu')_P$ is given by (7) with

$$\sigma_P = n_0 s_\nu \int_{x_P}^{\infty} e^{-2\lambda^2 R x} dx = \frac{n_c s_\nu}{2\lambda^2 R} e^{-\lambda^2 \xi^2}.$$

Writing $n_c s_\nu = \mu$, equation (10) becomes

$$E = \frac{\delta\omega}{4\pi} \mu H_\nu' \int_{-\infty}^{\infty} e^{-\lambda^2\xi^2} \left[2 + \frac{3\mu}{2\lambda^2 R} e^{-\lambda^2\xi^2} \right] \exp -\left\{ \frac{\mu\sqrt{\pi}}{2\lambda} (1 - \Phi(\xi\lambda)) \right\} d\xi. \quad (11)$$

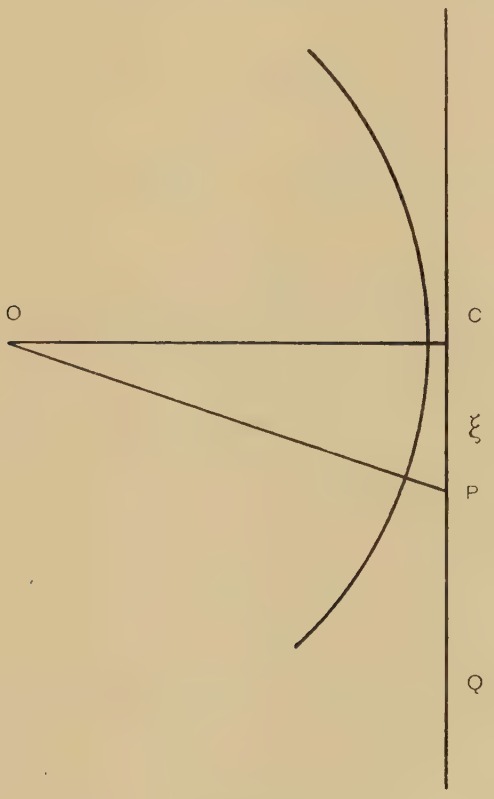

Fig. 4.

The term $\frac{3\mu}{2\lambda^2 R} e^{-\lambda^2\xi^2}$ may be neglected in comparison with 2. It is of course most important when μ is large, but when μ is large the term

$$\exp -\left\{ \frac{\mu\sqrt{\pi}}{2\lambda} (1 - \Phi(\lambda\xi)) \right\}$$

is overwhelmingly small unless $\lambda\xi$ is large enough to make $\Phi(\lambda\xi)$ close to unity, and when $\lambda\xi$ is large of course $\exp(-\lambda^2\xi^2)$ is small. The physical explanation of the legitimacy of neglecting this term is that when the absorption coefficient is large enough to cause appreciable banking of $\mathcal{J}_\nu'$

in $\mathfrak{J}_\nu' = H(2+3\sigma_\nu)$, self-absorption prevents us from seeing down to those layers in which the banking is appreciable.

Neglecting this term the efflux equation integrates as follows.

Let

$$Y = 1 - \Phi(\lambda\xi).$$

Then

$$dY = \frac{2}{\sqrt{\pi}} e^{-\lambda^2\xi^2} \cdot \lambda d\xi.$$

Equation (11) becomes

$$E = \frac{\delta\omega}{4\pi} \cdot \mu H_\nu' \int_0^2 \frac{\sqrt{\pi}}{\lambda} \exp\left(-\frac{\mu Y\sqrt{\pi}}{\lambda}\right) dY$$

$$= \frac{\delta\omega}{4\pi} H_\nu' (1 - e^{-2\mu\sqrt{\pi}/\lambda}). \qquad (12)$$

This expression has been given by Dyson and Woolley * derived in a different manner.

Notice that as μ increases E tends to the value $\frac{\delta\omega}{4\pi} H_\nu'$. The intensity of the emission line at the base of the chromosphere is accordingly equal to the central intensity of the Fraunhofer line (averaged over the disc). As proved here the result is trivial, because H_ν' is very nearly zero at the centre of the line. In this case even a high value of ϵ does not save the centre of the line from being blackened, as the chromosphere will produce a central absorption in any case.

8. Where the line of sight cuts the disc, the efflux given by the disc itself is given by equation (4),

$$E = \frac{\delta\omega}{4\pi}\left\{2H + \frac{(1-\epsilon)\eta A}{1+\eta}\right\}\{1 - \exp - \overline{1+\eta}\tau_1\}, \qquad (13)$$

where now A is given by equation (9). This efflux has to pass through the upper layer or chromosphere. Taking into account the chromosphere both behind and in front of the disc, we have, writing ζ for $\frac{\mu\sqrt{\pi}}{\lambda}$,

$$E = \frac{\delta\omega}{4\pi}\left[\left\{2H + \frac{(1-\epsilon)\eta A}{1+\eta}\right\}\{1 - \exp - \overline{1+\eta}\tau_1\}e^{-\zeta} + 2H_\nu'(1-e^{-\zeta})\right.$$
$$\left.\{1 + \exp - (\zeta + \overline{1+\eta}\tau_1)\}\right]. \qquad (14)$$

Equation (14) includes (12), which can be derived from it by putting $\tau_1 = 0$. It also includes (4), obtained by putting $\zeta = 0$.

A composite atmosphere can also be constructed having $(1+\eta)^{-1} = \alpha + \beta\tau$ in the lower layer. The equation for A is

$$-\frac{AK_0(x_0)}{H} = \frac{2 + \beta - \alpha(2+3\sigma_1)}{1 - \sqrt{\tfrac{1}{3}\alpha} \cdot K_0'(x_0)/K(x_0)}, \qquad x_0 = (12\alpha)^{\frac{1}{2}}\beta^{-1}, \qquad (9')$$

* *Eclipses of the Sun and Moon*, p. 86 (Oxford, 1937).

the blackening r for the mean contour being

$$\frac{H_\nu'}{H} = r = \frac{\alpha - (2+\beta)\sqrt{\tfrac{1}{3}\alpha}\,.\,K_0'(x_0)/K(x_0)}{1 - (2+3\sigma_1)\sqrt{\tfrac{1}{3}\alpha}\,.\,K_0'(x_0)/K(x_0)}.$$

The equation analogous to (5) and (14) for the efflux is

$$E = \frac{\delta\omega}{4\pi}[(2H + (1-\alpha)\{\beta H + AK_0(x_0)\})\{1 - \exp - \sqrt{1+\eta}\tau_1\}e^{-\zeta} + 2H_\nu'(1-e^{-\zeta})$$
$$(1 + \exp - (\zeta + \sqrt{1+\eta}\tau_1))]. \quad (14')$$

Equation (14′) includes as special cases (5) (by putting $\zeta = 0$) and (12) (by putting $\tau_1 = 0$). Inspection of (5) and (6) shows how to insert the C term, if wanted.

In the case where $\epsilon = 0$, equations (14) and (14′) do not give results which are markedly different from those given by (4) and (5) except that they show a definite centrally reversed chromospheric emission beyond the limb. Of course the results are quantitatively different, but it does not seem worth troubling the reader with an account of numerical work (one set of contours computed from (14) is, however, shown in fig. 5, *a*). Since we cannot attribute a high value of ϵ to collisions but to fluorescence, it is incorrect to insert a high value of ϵ with (14), because one would expect more fluorescence in the upper layers, where there is now no allowance for it, than in the lower layers.

It is to be noted that equations (14), etc. refer to the contour obtained when an infinitely narrow slit is used. Where there is a wide slit or where bad seeing confuses the observation, the contour to be expected is obtained by averaging (14) in a suitable manner with τ_1.

The next section gives a discussion of a mathematical model in which there is some attempt to represent fluorescence in the upper layers.

9. We now come to the introduction of some term in the equations of the upper atmosphere similar to the term ϵ in the lower atmosphere. The term ϵ was originally intended to represent the effect of collisions, which one would not attribute to the low pressure upper atmosphere. We are, however, concerned with the search for a mathematical representation of the observed behaviour of the lines, and we may find that the ϵ term is useful as representing to a first approximation those mechanisms which the theory with which we started explicitly avoids, to wit fluorescence. The simplest way of introducing a pseudo-collision or fluorescent term into the upper atmosphere is to suppose that the emission into the line is proportional to H. Let it be $l_\nu \rho f H$. Then the equation of transfer is

$$\cos\theta \frac{d\mathcal{J}_\nu'(\theta)}{\rho dx} = -l_\nu \mathcal{J}_\nu'(\theta) + l_\nu f H, \quad (15)$$

of which the solution for the radiation averaged in all directions similar to Eddington's system of approximations is

$$\mathcal{J}_\nu' - fH = K\cosh\sigma\sqrt{3} + L\sinh\sigma\sqrt{3}. \quad (16)$$

The boundary conditions are $\mathcal{J}_\nu' = 2H_\nu'$ at $\sigma = 0$, and the continuity of $\mathcal{J}_\nu'$ and H_ν' at the intersurface $\tau = 0$ and $\sigma = \sigma_1$. The three conditions are:

$$\left.\begin{aligned} 2H + A &= fH + K\cosh\sigma_1\sqrt{3} + L\sinh\sigma_1\sqrt{3} \\ 3H - qA &= \sqrt{3}\,.\,(1+\eta)\{K\sinh\sigma_1\sqrt{3} + L\cosh\sigma_1\sqrt{3}\} \\ 0 &= fH + K - \frac{2}{\sqrt{3}}L \end{aligned}\right\}. \tag{17}$$

The blackening of the mean contour is given by

$$\frac{H_\nu'}{H} = \frac{1}{\sqrt{3}}\frac{L}{H}, \tag{18}$$

and the explicit solution for A/H from (17) is

$$\begin{aligned} -A\left\{\cosh\sigma_1\sqrt{3} + \tfrac{2}{3}\sinh\sigma_1\sqrt{3} + \frac{q}{\sqrt{3}\,.\,(1+\eta)}\left(\sinh\sigma_1\sqrt{3} + \frac{2}{\sqrt{3}}\cosh\sigma_1\sqrt{3}\right)\right\} \\ = H\left\{f + (2-f)\left(\cosh\sigma_1\sqrt{3} + \frac{2}{\sqrt{3}}\sinh\sigma_1\sqrt{3}\right)\right. \\ \left. - \frac{\sqrt{3}}{1+\eta}\left(\sinh\sigma_1\sqrt{3} + \frac{2}{\sqrt{3}}\cosh\sigma_1\sqrt{3}\right)\right\}. \end{aligned} \tag{19}$$

While the solution of (15) for the radiation averaged in all directions is (16), the solution for any particular direction is

$$\mathcal{J}_\nu'(\theta) - fH = C\exp - \int l_\nu\rho\sec\theta\,d\theta,$$

the constant C being given by the efflux at the intersurface. After a little reduction we get

$$\begin{aligned} E = \frac{\delta\omega}{4\pi}\left\{2H + \frac{(1-\epsilon)\eta A}{1+\eta}\right\}\{1 - \exp - (1+\eta)\tau_1\}e^{-\zeta} + fH(1-e^{-\zeta}) \\ (1 + \exp - (\zeta + (1+\eta)\tau_1)), \end{aligned} \tag{20}$$

where A is given by (19) and the mean contour is given by (18).

We may add for completeness the equations when $(1+\eta)^{-1} = \alpha + \beta\tau$ in the lower atmosphere. The equations for A/H are:

$$\left.\begin{aligned} (2+\beta)H + AK_0(x_0) &= fH + K\cosh\sigma_1\sqrt{3} + L\sinh\sigma_1\sqrt{3} \\ \alpha H + \sqrt{\tfrac{1}{3}\alpha}\,.\,AK_0'(x_0) &= (\sqrt{3})^{-1}\{K\sinh\sigma_1\sqrt{3} + L\cosh\sigma_1\sqrt{3}\} \\ 0 &= fH + K - \frac{2}{\sqrt{3}}L \end{aligned}\right\}. \tag{17'}$$

The mean contour is still given by (18), but the efflux equation is

$$\begin{aligned} E = \frac{\delta\omega}{4\pi}[(2H + (1-\alpha)\{\beta H + AK_0(x_0)\})\{1 - \exp - (1+\eta)\tau_1\}e^{-\zeta} \\ + fH(1-e^{-\zeta})(1 + \exp - (\zeta + (1+\eta)\tau_1))]. \end{aligned} \tag{20'}$$

Special interest centres in the central intensity: as might be expected, just as by choosing a suitable value of ϵ in (4) we can get a high central intensity for the contour of a line at the limb, so by choosing a suitable value of f in (20) we can compute a contour for a chromospheric emission line which is not centrally reversed. In fact, equations (20) and (14) correspond to ϵ non-zero and ϵ zero in equation (4). But in view of the observed low central intensities of metallic lines on the disc, we must inquire whether (20) succeeds in giving an unreversed chromospheric emission line without making the central intensity of the line on the disc too high.

When η is large, (18) and (19) reduce to

$$r = \frac{f}{2 + \sqrt{3} \,.\, \coth \sigma_1 \sqrt{3}},$$

while (20) reduces to

$$E = \frac{\delta\omega}{4\pi} \,.\, fH(1 - e^{-\zeta}).$$

Now $\zeta = \lambda^2 R \sqrt{\pi} \,.\, \sigma_1$, so that a fair estimate for the chromosphere is $\zeta = 27\sigma_1$. Suppose $\sigma_1 = 0{\cdot}12$ and $\zeta = 3{\cdot}1$. Then with $f = \frac{1}{2}$ we find $r = 0{\cdot}05$ and $E = \frac{\delta\omega}{4\pi} H \times 0{\cdot}48$. Since the intensity on the disc is $\frac{\delta\omega}{4\pi} H(2 + 3\cos\theta)$, the intensity of the base of the chromosphere is 0·24 times the intensity of the continuous spectrum at the limb. The central intensity of the line at the extreme limb is also 0·24. It would be easier to obtain a high intensity in the chromosphere together with a low central intensity on the disc if f increased outwards instead of being constant.

Fig. 5 illustrates equations (14) and (20). The set (*a*) shows contours computed from (14) with $\eta = 300\sigma_1$ and $\epsilon = 0$; the distinctive feature of this equation, which is the case where there is no fluorescence, is the heavy central reversal of the chromospheric line. Set (*b*) shows equation (20) with $\eta = 300\sigma_1$, $\epsilon = 0$ and $f = 1$. The contours resemble the Kamishari results for ionised lines, but it must be noted that the central intensity of the line on the disc is about 20 per cent., so that this amount of fluorescene is not permissible in the case of lines whose central intensities are known to be low—presumably the great majority of metallic lines. Set (*c*) shows $\eta = 20{,}000\sigma_1$, $\epsilon = 0$ and $f = 1/2$. There are far less atoms in the chromosphere than in the two previous cases, and the central intensity on the disc is down to 5 per cent., but the chromospheric emission is not reversed as in (*a*).

For various reasons it does not seem worth while to attempt to make quantitative comparisons with the Kamishari contours, but it seems highly desirable that the observations should be repeated at future eclipses with instruments of still higher resolving power. With accurately observed contours it will be worth while examining numerical applications of the formulæ developed above, and perhaps to refine the theory, which has been given only its first approximate form.

10. Equation (14) or (20) can be used to predict the emission gradient of the chromosphere in terms of the density gradient. A remarkable

Fig. 5.—(a) *Contours computed from equation* (14) *with* $\eta = 3000\sigma_1$ *and* $\epsilon = 0$. (b) *Contours computed from equation* (20) *with* $\eta = 3000\sigma_1$, $\epsilon = 0$ *and* $f = 1$.
(c) *Contours computed from equation* (20) *with* $\eta = 20{,}000\sigma_1$, $\epsilon = 0$ *and* $f = \frac{1}{2}$.

consequence of equation (20) is that it removes part at least of the well-known difficulty in reconciling the observed emission gradient with the density gradient under gravitational equilibrium.

It is usual to treat the emission gradient as if it could be converted into a density gradient without further ado : in other words, self-absorption is neglected, and the intensity of the light received by the spectroscope is taken to be proportional to the number of atoms along the line of sight, and therefore (see § 7, $N \propto n_c$) to the number of atoms per c.c. on the radius vector normal to the line of sight. When observations are interpreted in this way it is found that the density gradient is smaller than would be the case for an atmosphere in isothermal gravitational equilibrium. An actual departure from the gravitational density gradient is in fact generally accepted, and explanations have been looked for based on selective radiation pressure, on turbulence, and on electrical forces.*

The observed emission gradient for H_γ according to Pannekoek and Minnaert † is

$$I = I_0 \exp - \frac{H}{617},$$

the height H being reckoned in km. If the atmosphere has an atomic weight μ, the density gradient for a uniform temperature of 6000° is

$$\rho = \rho_0 \exp - \frac{H}{180}\mu.$$

But according to (20), $I = \int E_\nu d\nu$ is not proportional to ρ but to the square root of ρ if f is constant (it is supposed that $s_\nu \propto (\nu - \nu_0)^{-2}$). We have, in fact,

$$I = I_0 \exp - \frac{H}{360}\mu.$$

To estimate μ we have to rely on Unsöld's estimate ‡ that the partial pressure of Ca^+ at the base of the chromosphere is $2 \cdot 10^{-15}$ atmospheres. The metals are then nearly all twice ionised, and hydrogen is about seven-tenths ionised. If we suppose there are 61 hydrogen atoms to every metallic atom, we find for μ, counting in the electrons, $\mu = 0{\cdot}87$. Recent estimates of the number of hydrogen atoms per metallic atom are greater, giving lower values of μ.

With $\mu = 0{\cdot}87$, we have

$$I = I_0 \exp - \frac{H}{410}.$$

In view of the uncertainty attached to μ we can hardly say that there is any substantial disagreement with the observed figure, especially as the higher estimates of hydrogen content tend to increase the denominator and close

* *Cf.* Dyson and Woolley, *Eclipses of the Sun and Moon*, chaps. xi and xii.

† *Verh. K. Akad. Wet. Amsterdam*, **13**, 1928.

‡ *Mt. Wilson Contr.* 377 ; *Ap. J.*, **69**, 209, 1929.

the gap between theoretical and observed figures. Moreover, we should expect f to increase outwards, which also tends to increase the denominator 410.

Some effect is given by equation (14), though not so striking as that given by (20). By integrating curves from (14) with a planimeter, I find that it gives I proportional to $\rho^{0\cdot8}$.

11. The topic of lines at the limb recalls Julius's theories of anomalous dispersion.* These theories have been abandoned by general consent nowadays, and there is no reference to them that I have been able to find in any modern textbook on astrophysics. They are given prominence in R. W. Wood's *Physical Optics*.† Originally framed to account for the flash spectrum by analogy with a laboratory experiment performed by Wood, Julius developed his theory to account for a great many solar phenomena. Most of the developments have been demonstrated to be untenable,‡ but it is difficult to avoid a feeling that anomalous dispersion may after all have some observable effects at the limb. As I have not been able to find a formal proof that it may in fact be ignored, I venture to suggest one.

It is well known that in the neighbourhood of an absorption line both the refractive index and the absorption coefficient of a gas undergo rapid changes. The changes in the absorption coefficient are well recognised—they are, in fact, what we have been assuming tacitly in preceding sections, *i.e.* that η is a function of ν. Nothing has been said about refraction. In the refracting medium the wave-length and velocity of light are both diminished, and a coherent beam possessing a definite wave front is refracted through a definite angle if it encounters a density gradient or passes through a prism-shaped mass of the material. When, however, light is scattered the wave front is broken up and there is no refraction. The distinction is that between a stellar image seen through a prism-shaped prominence, which would be displaced, and a general flux of radiation passing through the refracting medium. In the latter case the criterion whether the refracting power of the inhomogeneous material exercises any influence on the flow of radiation is whether a beam of light can proceed along a distance sufficiently large to bend it appreciably before it is scattered. We have experimental evidence as well as theoretical reasons for believing that an atom which has absorbed a quantum of radiation delays a matter of 10^{-8} sec. before re-emitting the quantum, so that even if the quantum is sent on in exactly the same direction as it was going before it was stopped, the wave front has been broken by the scattering.

To examine the case of the chromosphere we calculate the angle through which a beam is bent when proceeding parallel to the Sun's surface for a distance such that the beam is reduced to $1/e$ of its intensity.

The refractive index and absorption coefficient of a gas consisting of bound electrons considered as harmonic oscillators is given by classical

* Julius, *Ap. J.*, **12**, 195, 1900; **40**, 1, 1914; **43**, 43, 1916. Einstein, *Ap. J.*, **63**, 196, 1926.

† 2nd edition, p. 115.

‡ St. John, *Ap. J.*, **41**, 28, 1915; **44**, 311, 1916. Brunt, *M.N.*, **73**, 568, 1913.

electromagnetic theory. In quantum theory an atom is represented by an oscillator strength f of classical oscillators. For definiteness we will here set $f=1$. Then the refractive index μ_ν and scattering coefficient s_ν for a rarefied atmosphere are given by *

$$\mu_\nu = 1 + \frac{Ne^2(\nu-\nu_0)}{4\pi m\nu_0\{(\nu-\nu_0)^2+\gamma^2\}},$$

$$s_\nu = \frac{e^2}{mc}\frac{\gamma}{(\nu-\nu_0)^2+\gamma^2}.$$

Here there are N oscillators per c.c. and $\gamma = 2\pi e^2\nu_0^2/3mc^3$.

Now consider two rays travelling parallel to the Sun's surface at heights h and $h+\Delta h$ above some datum. The retardation of the wave front of the lower beam after a distance L has been traversed is

$$-L\frac{dN}{dh}\Delta h\frac{e^2}{4\pi m\nu_0}\frac{(\nu-\nu_0)^2}{(\nu-\nu_0)^2+\gamma^2}$$

$$= -\frac{dN}{dh}\Delta h \,.\, Ls_\nu \,.\, \frac{\nu-\nu_0}{\nu_0}\,.\,\frac{c}{4\pi\gamma}.$$

The beam will be reduced to $1/e$ of its original strength when $Ls_\nu = 1$. The angle through which the beam is refracted is then

$$\theta = -\frac{dN}{dh}\frac{\nu-\nu_0}{\nu_0}\frac{c}{4\pi\gamma}.$$

(1) If we assume that the refraction may be neglected we may make use of current estimates of the density gradient of the chromosphere, and set $\frac{dN}{dh} = -\Lambda$, where $\Lambda = \frac{1}{3}\,.\,10^{-8}$ cm.$^{-1}$. Now $4\pi\gamma$ is of order 10^8, so that if we set $(\nu-\nu_0)/\nu_0 = 10^{-4}$ corresponding to 0·6 A. from one of the D lines we get $\theta = 10^{-10}$ and refraction is utterly negligible. The view that refraction is negligible is therefore self-consistent.

(2) If θ is $1'$, Λ must be about 10^{-2}. Most of the chromosphere is contained in one metre. But there is no physical agency which could conceivably compress it so much, and the position is untenable.

In the laboratory experiment on *Na* vapour, on the other hand, the density gradient is much higher. Suppose the density of (sodium) vapour falls to e^{-1} in ascending a height of 3 mm. Then $\Lambda = 10/3$, giving $\theta = 6°$ approximately.

Refraction effects are only observable in the Earth's atmosphere—which has a density gradient much greater than that of the chromosphere—when the air path is very long, and there can be no anomalous dispersion with such a weak gradient because the high scattering coefficient which of necessity accompanies the anomalous dispersion breaks up coherent beams before they can experience appreciable refraction.

* *Cf.* Rosseland, *Theoretical Astrophysics*, p. 94.

Summary.—Theoretical contours of lines changing over from absorption to emission at the extreme limb are calculated from a strict application of the models η constant and $(1+\eta)^{-1} = \alpha + \beta\tau$, and also from equations intended to represent fluorescence in an approximate way. It is suggested that further eclipse observations of spectra at the extreme limb will throw light on the theory of the formation of lines : in particular *the chromospheric emission lines should show reversed contours if there is no fluorescence* or any other failure of simple theory. The contours for chromospheric lines show that part of the discrepancy between the observed emission gradient of the chromosphere and the density gradient given by gravitational equilibrium may be attributable to self-absorption. In the concluding section (§ 11) it is shown that anomalous dispersion can have no appreciable effect in the chromosphere.

STELLAR LUMINOSITY AND THE OPACITY IN THE OUTER LAYERS OF A STAR.

E. A. Milne.

(Communicated by the Director of the University Observatory, Oxford)

1. In a recent paper entitled "Stellar Luminosity and Photospheric Pressure" I obtained a relation between the luminosity of a star, its mass, its mean extra-photospheric opacity and its photospheric molecular weight, in a form involving a numerical coefficient dependent on the density-distribution in the interior.* The essential step in the calculation was to determine the pressure at the photospheric level by two distinct methods, one by working outwards from the stellar centre, one by working inwards from the stellar boundary; the equating of the two values gave the relation in question. This use of the continuity of pressure is capable of considerable extension; we can apply it to other levels than $\tau = \frac{2}{3}$, $T = T_1$. In the present paper I develop its application to levels lying far underneath the photosphere, but yet sufficiently far from the centre of the star to be counted in the "boundary" layers. In this region the effect of curvature may still be neglected, the acceleration due to gravity has effectively its surface value, the included mass is effectively the total mass M of the star and the total net flow of radiation is effectively the luminosity L of the star. (These approximations have been used by many writers.) In symbols,

$$\frac{r_1 - r}{r_1} << 1, \qquad g \sim g_1, \qquad M(r) \sim M, \qquad L(r) \sim L,$$

in the notation used in paper I.† But the gas-pressure p, density ρ, temperature T and possibly opacity κ will in this region range over values considerably different from their photospheric values. It is our object to analyse the equilibrium of this layer by considering the continuity of gas-pressure and to trace the fit of the layer to the deeper interior.

2. *Equilibrium of the Interior.*—We begin by generalising some results of the previous paper, using the same notation. As in that paper, the mechanical equilibrium of the interior is expressed by an equation

$$\frac{1}{x^2}\frac{d}{dx}\left[\frac{x^2}{y}\frac{dz}{dx}\right] = -y \qquad (1)‡$$

where

$$y = \rho/\rho_c, \qquad z = P/P_c, \qquad x = \left[\frac{4\pi G \rho_c^2}{P_c}\right]^{\frac{1}{2}} r, \qquad (2)§$$

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* *M.N.*, **97**, 513, 1937. Referred to as I.

† Many of the formulæ which follow are also valid if curvature is taken into account.

‡ I, equation (5).

§ I, equations (3), (4).

P being the total pressure. Equation (1) becomes a differential equation in y on taking $z = f(y)$ where $f(y)$ is such that $f(1) = 1$, $f(0) = 0$; $f(y)$ is descriptive of the march of total pressure P with density ρ in the interior, and so is descriptive of the compressibility of the interior. The resulting differential equation is to be solved with the initial conditions :

$$\text{at } x = 0, \qquad y = 1, \qquad \frac{dy}{dx} = 0. \tag{3}$$

This is the procedure ordinarily followed in analysing interior equilibrium, some physical assumptions being made which determine $f(y)$. For example, if, as in many cases, the physical assumptions lead to some kind of polytropic equilibrium, we have just $z = y^{1+1/n}$, where n is the polytropic index, and the parametric substitutions

$$y = \theta^n, \qquad z = \theta^{n+1}, \qquad x = (n+1)^{\frac{1}{2}}\xi \tag{4}$$

convert (1) into Emden's equation of index n,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d\theta}{d\xi}\right] = -\theta^n. \tag{5}$$

The solution $y = y(x)$ selected by conditions (3) will usually have a first zero at some point $x = x_0$. If the solution were followed through to this point, the corresponding configuration would satisfy the boundary conditions

$$\rho_0 = 0, \qquad P_0 = 0, \qquad T_0 = 0.$$

As $T_0 = 0$ does not correspond to a real star, we have to abandon this solution before we get to the zero $x = x_0$. We then switch over to another solution appropriate to the layers in which the effect of the true boundary condition $T_0 = 2^{-\frac{1}{4}}T_1 \neq 0$ becomes appreciable.* In the former paper we assumed $f(y)$ so chosen that it fitted the actual star as far out as $\tau = \frac{2}{3}$, $T = T_1$. We shall temporarily do the same again, but in the sequel we shall not need to make so drastic an assumption ; we shall see that, in the equations which are the main subject of this paper, it is not necessary to make a choice of $f(y)$ beyond the layers in which the effect of the non-zero boundary temperature is inappreciable ; in other words, $f(y)$ need only be chosen so as to be appropriate to "interior" conditions.

Associated with (1) and (2) we have a formula for the included mass of the form

$$M(r) = \frac{P_c^{\frac{3}{2}}}{(4\pi)^{\frac{1}{2}}G^{\frac{3}{2}}\rho_c^2}\left[-\frac{x^2}{y}\frac{dz}{dx}\right]. \tag{6}$$†

In this we substitute for ρ_c and P_c from (2), obtaining

$$M(r) = \frac{P^{\frac{3}{2}}}{(4\pi)^{\frac{1}{2}}G^{\frac{3}{2}}\rho^2}\frac{y}{z^{\frac{3}{2}}}\left[-x^2\frac{dz}{dx}\right]. \tag{7}$$

This is apparently a retrograde step, but it makes further progress possible.

* T_1 is the star's effective temperature.

† I, equation (9).

Now apply this relation to any place sufficiently near the boundary of the star (though necessarily inside the photosphere) for the material there to be certainly gaseous. Let μ be the mean molecular weight at this point, T the actual temperature. Then

$$p=(R/\mu)\rho T. \tag{8}$$

Define a parameter β by

$$\beta=p/P. \tag{9}$$

The parameter β in general varies with depth. Eliminating ρ and P from (7) by means of (8) and (9) we find

$$M(r)=\frac{(R/\mu)^2T^2}{(4\pi)^{\frac{1}{2}}G^{\frac{3}{2}}\beta^{\frac{3}{2}}p^{\frac{1}{2}}}\frac{y}{z^{\frac{3}{2}}}\left[-x^2\frac{dz}{dx}\right].$$

This yields the gas-pressure p at this depth in the form

$$p=\frac{(R/\mu)^4T^4}{4\pi G^3\beta^3[M(r)]^2}\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2. \tag{10}$$

This is the value of p to which we are led by working *outwards* from the stellar centre. It contains mention only of quantities relating to the level considered and of quantities determined by conditions interior to this level. The numerical value of the coefficient

$$\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2 \tag{11}$$

is to be obtained by solving (1) starting from the centre with conditions (3), for some function $f(y)$. For example, if the interior is polytropic, by (4) we have

$$\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2=(n+1)^3\theta^{n-3}\left[-\xi^2\frac{d\theta}{d\xi}\right]^2, \tag{11'}$$

which can be obtained from the standard tables of Emden functions.

The power T^4 on the right-hand side of (10) has no connection with the radiation fourth-power law.

3. *Equilibrium of the Outer Layers.*—In these layers, in which curvature can be neglected and the included mass is practically M, we may take g as approximately constant and equal to its surface value. The mechanical equilibrium is then governed by

$$\frac{dP}{dr}=-g\rho, \tag{12}*$$

where $P=p+\frac{1}{3}aT^4$*. The previous relation (10) is independent of any assumption about radiative equilibrium, but we now definitely assume that the outer layers are in radiative equilibrium. In this case the temperature distribution is given by †

$$T^4=\tfrac{1}{2}T_1^4(1+\tfrac{3}{2}\tau), \tag{13}$$

* It was shown in Paper I that these equations hold up to the boundary.

† *M.N.*, **81**, 361, 1921 ; *Phil. Trans.*, **223**, A, p. 205, equation (2A), 1922.

where

$$\frac{d\tau}{dr} = -\kappa\rho \tag{14}$$

and κ is the coefficient of opacity, which will in general vary with depth. Hence, as usual,

$$\frac{d\tau}{dP} = \frac{\kappa}{g},$$

and so

$$\tau = \frac{1}{g}\int_{P_0}^{P} \kappa dP = \frac{P - P_0}{g}\bar{\kappa}, \tag{15}$$

where

$$\bar{\kappa} = \frac{1}{P - P_0}\int_{P_0}^{P} \kappa dP = \frac{\int_{r}^{r_0} \kappa\rho dr}{\int_{r}^{r_0} \rho dr}. \tag{16}$$

Thus $\bar{\kappa}$ is the mean opacity of the material *exterior* to the level considered. In general $\bar{\kappa}$ will also vary with depth.

Now, since $P_0 = \frac{1}{3}aT_0^4 = \frac{1}{6}aT_1^4$, we have

$$P - P_0 = p + \tfrac{1}{3}aT^4 - \tfrac{1}{6}aT_1^4,$$

and so by (15)

$$\frac{\tau g}{\bar{\kappa}} = p + \tfrac{1}{3}a(T^4 - \tfrac{1}{2}T_1^4). \tag{17}$$

But by (13)

$$\tau = \tfrac{4}{3}(T^4 - \tfrac{1}{2}T_1^4)/T_1^4, \tag{18}$$

and, moreover, since

$$L = \pi a c r_1^2 T_1^4 \tag{19}$$

we have

$$g = \frac{GM}{r_1^2} = \frac{\pi c GM}{L}aT_1^4. \tag{20}$$

Inserting (18) and (20) in (17) we have

$$p = \tfrac{1}{3}a(T^4 - \tfrac{1}{2}T_1^4)\left(\frac{4\pi c GM}{\bar{\kappa}L} - 1\right). \tag{21}$$

This is the value of p to which we are led by working *inwards* from the stellar boundary. It contains mention only of quantities relating to the level considered and of quantities determined by conditions exterior to this level.

Further, at this depth we now know β. For by (9) and (21)

$$\beta = \frac{p}{p + \frac{1}{3}aT^4} = \frac{\left(1 - \frac{\bar{\kappa}L}{4\pi c GM}\right)(T^4 - \frac{1}{2}T_1^4)}{T^4 - \frac{1}{2}T_1^4\left(1 - \frac{\bar{\kappa}L}{4\pi c GM}\right)}. \tag{22}$$

4. *Continuity of Pressure.*—The two values of p given by (10) and (21) must be equal when both equations relate to the same level. Equating them, putting $M(r) \sim M$, and replacing β by its value (22), we have

$$\tfrac{1}{3}a(T^4 - \tfrac{1}{2}T_1^4)\left(\frac{4\pi cGM}{\bar{\kappa}L} - 1\right) = \frac{(R/\mu)^4 T^4}{4\pi G^3 M^2} \frac{\left[T^4 - \tfrac{1}{2}T_1^4\left(1 - \dfrac{\bar{\kappa}L}{4\pi cGM}\right)\right]^3}{\left(1 - \dfrac{\bar{\kappa}L}{4\pi cGM}\right)^3 (T^4 - \tfrac{1}{2}T_1^4)^3} \frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2,$$

which may be re-written in the form

$$\bar{\kappa}L = \frac{16\pi^2 cG^4M^3(\frac{1}{3}a)}{(R/\mu)^4} \frac{\left(1 - \dfrac{\bar{\kappa}L}{4\pi cGM}\right)^4 \left(1 - \dfrac{\frac{1}{2}T_1^4}{T^4}\right)^4}{\left[1 - \dfrac{\frac{1}{2}T_1^4}{T^4}\left(1 - \dfrac{\bar{\kappa}L}{4\pi cGM}\right)\right]^3} \frac{z^3}{y^2}\left[-x^2\frac{dz}{dx}\right]^{-2}. \qquad (23)$$

5. *Division of the Boundary Layers into Zones.*—Equation (23) contains T explicitly only in the form $\frac{1}{2}T_1^4/T^4$. This suggests the following division of the outer parts of a star into zones :—

(*a*) *Zone A.*—This is defined as being the region above the photosphere, namely, $T_0 \leqslant T \leqslant T_1$. The equilibrium of this layer requires a special treatment which has been given by various writers. Relation (23) is not effectively applicable to this zone, since we have supposed $z = f(y)$ to fit the star at most up to $T = T_1$.

(*b*) *Zone B.*—This is defined as being that region immediately below $T = T_1$ where T^4 is still comparable with $\frac{1}{2}T_1^4$. It is the sub-photospheric or transition layer, where the effect of the non-zero boundary temperature is still appreciable. Equation (23) traces the behaviour in this zone, provided $z = f(y)$ is chosen to fit the star right up to the photosphere. In particular it then applies to the level $T = T_1$, the photosphere, yielding

$$\bar{\kappa}_1 L = \frac{8\pi^2 cG^4M^3(\frac{1}{3}a)}{(R/\mu_1)^4} \frac{\left(1 - \dfrac{\bar{\kappa}_1 L}{4\pi cGM}\right)^4}{\left(1 + \dfrac{\bar{\kappa}_1 L}{4\pi cGM}\right)^3} \frac{z_1^3}{y_1^2}\left[-x_1^2\left(\frac{dz}{dx}\right)_1\right]^{-2}, \qquad (24)$$

where $x = x_1$ corresponds to the level $T = T_1$. This is the relation obtained directly * in Paper I, and there applied. For known (observed) values of L, M, $\bar{\kappa}_1$, μ_1 it determines the value to which the dimensionless coefficient (11) must tend as we approach the photosphere from inside.

(*c*) *Zone C.*—This is defined as being the region in which T^4 is large compared with $\frac{1}{2}T_1^4$, but which is still so near the surface that we have effectively $r \sim r_1$, $M(r) \sim M$, $L(r) \sim L$. Roughly speaking, we may describe it as the region where T^4, though large compared with T_1^4, is small compared with T_c^4. For example, in the case of the Sun ($T_1 = 6000°$), we could take it as where T lies between 30,000° and 300,000°. It is the region where most writers on stellar interiors have placed the "effective boundary"

* I, equation (28).

of the star, *i.e.* the region up to and including which the assumptions made for the interior still approximately hold good.

Putting $\frac{1}{2}T_1^4/T^4 << 1$ in (23) we get in Zone C

$$\bar{\kappa}L = \frac{16\pi^2 cG^4M^3(\frac{1}{3}a)}{(R/\mu)^4}\left(1 - \frac{\bar{\kappa}L}{4\pi cGM}\right)^4 \frac{z^3}{y^2}\left[-x^2\frac{dz}{dx}\right]^{-2}. \tag{24'}$$

6. *Values of β in the Various Zones.*—The behaviour of β is given by (22). At the top of Zone A we have $T^4 = T_0^4 = \frac{1}{2}T_1^4$, and

$$\beta = 0.$$

At the top of Zone B we have $T = T_1$, and hence $\beta = \beta_1$, where

$$\beta_1 = \frac{1 - \dfrac{\bar{\kappa}_1 L}{4\pi cGM}}{1 + \dfrac{\bar{\kappa}_1 L}{4\pi cGM}} \tag{25}$$

or

$$\bar{\kappa}_1 = \frac{4\pi cGM}{L}\,\frac{1-\beta_1}{1+\beta_1}. \tag{26}$$

In Zone C, where $\frac{1}{2}T_1^4/T^4 << 1$, we have by (22)

$$\beta = 1 - \frac{\bar{\kappa}L}{4\pi cGM} \tag{25'}$$

or

$$\bar{\kappa} = \frac{4\pi cGM}{L}(1-\beta). \tag{26'}$$

7. Formula (26′) for the mean exterior opacity down to Zone C is well known; it is given, in particular, by B. Strömgren,* and it may in fact be derived directly, without neglecting the variation of r in Zone C, from the standard equations

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho, \qquad \frac{dp'}{dP} = -\frac{\kappa\rho}{c}\,\frac{L}{4\pi r^2}$$

on eliminating r, using boundary conditions and taking $\frac{1}{2}T_1^4/T^4 << 1$. $\bar{\kappa}$ is still the mean *exterior* opacity, defined here by

$$\bar{\kappa} = \frac{1}{P - P_0}\int_{P_0}^{P} \kappa dP.$$

The novelty of the present investigation consists, not in the use of (26′) by itself, but in showing that the proper value of β to use in (26′) is precisely that given by the usual quartic equation, in which the value of the molecular weight μ occurring is *not* a mean through the interior, but the actual molecular weight at the level to which (26′) applies. This we now show.

* *Handb. d. Astrophys.*, Bd. 7, p. 169, equation (147).

8. *The Quartic Equation.*—What we really do is to put equations (24), (24′) in a parametric form, by use of β. By (22), at any level, the pressure p is given by

$$p = \frac{\beta}{1-\beta}\tfrac{1}{3}aT^4.$$

This must equal the value given by (10). Hence at any level

$$\frac{\beta}{1-\beta}\tfrac{1}{3}aT^4 = \frac{(R/\mu)^4 T^4}{4\pi G^3\beta^3[M(r)]^2}\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2.$$

Hence in Zones B or C, setting $M(r) \sim M$, we have

$$\frac{\beta^4}{1-\beta} = \frac{(R/\mu)^4}{4\pi G^3 M^2(\frac{1}{3}a)}\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2. \tag{27}$$

If in (27) we insert for β from (25) we recover (24); if in (27) we insert for β from (25′) we recover (24′).

9. In Zone C, (27) is to be used in connection with (26′). Relation (27) is very general; it assumes perfect-gas conditions in Zone C, but it does not assume radiative equilibrium, and it assumes nothing about the interior save that it is in mechanical equilibrium. The ratio β of gas-pressure to total pressure may vary in any manner through the star, but the value given by (27) is that appropriate to (26′); and the value of μ occurring in (27) is that for the level to which (26′) applies. Thus (27) and (26′) together determine the value of $\bar{\kappa}$ at the level in Zone C to which they apply—that is, the mean opacity *exterior* to this level.

10. *Special Case. The Polytrope* $n=3$.—Formula (27), the quartic equation for β, only becomes definite when some assumptions for the interior are made, sufficient to determine the numerical value of the coefficient (11), which occurs in (27). To illustrate the use of the formulæ, we shall now for the first time make an assumption about the "interior"—that is, about Zone C and the layers inside it. Suppose that the density-distribution in this "interior" is that of a polytrope $n=3$. This is known to be so when (1) the entire star is gaseous of constant molecular weight μ, (2) the state is one of radiative equilibrium throughout and (3) $\kappa\eta$ = constant, where $\eta = [L(r)/L]/[M(r)/M]$. It could also be so under other sets of assumptions. We do not need the detailed assumptions, but only the form of the density-distribution. Then by (11′), since $n=3$,

$$\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2 = (3+1)^3\left[-\xi^2\frac{d\theta}{d\xi}\right]^2,$$

where θ is the E-solution of Emden's equation of index 3. In Zone C ξ is inappreciably different from ξ_0, the first zero of $\theta(\xi)$, and hence (27) takes the form

$$\frac{\beta^4}{1-\beta} = \frac{(R/\mu)^4}{4\pi G^3 M^2(\frac{1}{3}a)}(3+1)^3\left[-\xi_0^2\left(\frac{d\theta}{d\xi}\right)_0\right]^2, \tag{28}$$

where

$$-\xi_0^2\left(\frac{d\theta}{d\xi}\right)_0 = 2{\cdot}018. \ . \ . \ . \qquad (29)$$

Equation (28) is precisely Eddington's quartic equation for β, with his value of the numerical coefficient. The root β of this equation, introduced into (26′), determines the mean opacity $\bar{\kappa}$ of the exterior layers of the star. Equation (26′) gives the value to which, for an interior $n=3$, the exterior mean opacity must settle down in Zone C in order that this zone shall be in equilibrium and shall be transmitting radiation across itself at the net rate L. The value of β in the interior may vary in any manner in different cases, subject to the interior remaining a polytrope of index 3, but equation (28) always determines the value of β to use in (26′), *i.e.* the value which gives the mean *exterior* opacity.

We can, further, determine the mean extra-photospheric opacity $\bar{\kappa}_1$ if the polytropic state extends through the layers of Zone B to the photosphere $T=T_1$. For then the root of (28) is also β_1, and so, by (25) and (25′),

$$1-\frac{\bar{\kappa}L}{4\pi cGM}=\frac{1-\dfrac{\bar{\kappa}_1 L}{4\pi cGM}}{1+\dfrac{\bar{\kappa}_1 L}{4\pi cGM}}$$

or

$$\frac{\bar{\kappa}}{\bar{\kappa}_1}=\frac{2}{1+\dfrac{\bar{\kappa}_1 L}{4\pi cGM}}, \qquad \frac{\bar{\kappa}_1}{\bar{\kappa}}=\frac{1}{2-\dfrac{\bar{\kappa}L}{4\pi cGM}}.$$

This determines the value of $\bar{\kappa}_1$ which the star must have in order that the radiation transmitted to space across the photosphere shall be L, on the polytropic assumption $n=3$.

In what follows we shall not use the last result, but shall limit ourselves to the form of the density-distribution in and interior to Zone C, thus not considering the form of $f(y)$ in Zone B.

11. It will be seen that we have now solved an outstanding problem: when β is determined by the quartic equation (28), what is the opacity given by the right-hand side of (26′)? The answer is given by (16); the right-hand side of (26′) gives the mean opacity $\bar{\kappa}$ of the *exterior* layers of the star, as evaluated down to Zone C. It does not in any way provide information about the interior opacity. The latter is irrelevant in this problem; equation (26′) provides the value of the mean exterior opacity whenever the interior is arranged as a polytrope of index 3.

Corresponding to any assumed density-distribution and any law of opacity-variation in the interior, we can always find a source-distribution compatible with radiative equilibrium.* We have then a definite model. Assuming gas-conditions and using the law of opacity, we may extrapolate if we wish from the opacity in Zone C to the opacity at the centre of the

* *M.N.*, **90**, 29 and 31, 1929.

star, assuming, of course, that one and the same opacity-law is applicable all the way from Zone C to the centre. But such calculations are pure extrapolations—they do not lend themselves to any independent check or make new inferences; they amount simply to the hypothesis that there are no unsuspected changes in ultimate chemical composition on the way to the centre, for example, but they do not provide any evidence for their correctness. The possibility of such calculations must not be allowed to obscure the fact that (28) and (26′) convey information only about the mean *exterior* opacity $\bar{\kappa}$, and that the relevant molecular weight μ in (28) is that in Zone C. Consequently, if (after making the hypothesis $n=3$) we adjust μ so as to reconcile the value $\bar{\kappa}$ determined "astronomically" by (28) and (26′) with that determined "physically" by direct calculation, we gain information only about the chemical composition of the exterior layers of the star. To suppose that (26′) gives about $2{\cdot}5\ \kappa_c$, as in Eddington's solution, may or may not be empirically satisfactory, and may indeed be a correct extrapolation to the centre on particular hypotheses, but it is misleading; κ_c is not involved at all. Similarly the current inference that the interiors of stars are largely composed of hydrogen, reached by adjustment of μ, seems without foundation. It is a mere extrapolation from the state of affairs in the exterior layers, to which the calculations really refer. If the extrapolations are consistently carried out, the result of the calculation holds good, not for the interior (which has played the part of a dummy), but for the exterior layers.

12. *Other Polytropes.*—We must now examine generally the effect of making other assumptions about the internal density-distribution, and enquire whether there is any possible observational check on any such assumption.

The assumption $n=3$ leads to a definite value for the necessary opacity in Zone C. For mathematical consistency this is found ultimately to require a physical law of opacity in Zone C of the form $\kappa \propto \rho/T^3$. It was kindly pointed out to me by Professor H. N. Russell in correspondence that if actually $\kappa \propto \rho/T^{\frac{7}{2}}$, then the relation (24′) already given determines the polytropic index in Zone C as $3\frac{1}{4}$—a case of a result found under different circumstances by Jeans.* Following Russell's idea, I shall consider the case $\kappa \propto \rho/T^\sigma$; to avoid interrupting the march of the argument which follows I shall relegate certain self-contained calculations to the appendix.

The next simplest type of assumption as to internal density-distribution after $n=3$ is to suppose that Zone C and the layers underneath form an almost complete polytrope of some index n. Let us examine the consequences. In this case the factor θ^{n-3} in (11′), which occurs as an inverse on the right-hand side of (24′), does not disappear. Then, as shown by Russell, the variation of the factor θ^{3-n} throughout Zone C must, according to (24′), be taken up by the variation of $\bar{\kappa}$ in Zone C. For all except very luminous stars, that is, unless $\bar{\kappa}L/4\pi cGM$ is appreciable compared with unity, this means that in Zone C we must have $\bar{\kappa} \propto \theta^{3-n}$. Now by direct

* *Astronomy and Cosmogony* (1928), p. 88.

integration inwards it is readily found that in Zone C $\bar{\kappa}$ behaves according to the formula *

$$\bar{\kappa} = \left[\frac{4\pi cGM\kappa_0(\frac{1}{3}a)}{(R/\mu)L}\,\frac{\sigma+5}{8}\,\frac{1}{T^{\sigma-3}}\right]^{\frac{1}{2}}, \tag{30}$$

provided that $\sigma > 3$. (Here κ_0 is the constant of proportionality in the formula $\kappa = \kappa_0\rho/T^\sigma$). Further, *without assuming anything about the gas-laws in the interior*, it can be shown that for a gaseous Zone C the temperature T is connected with the Emden variable θ by the relation

$$\frac{R}{\mu}\,\frac{T}{\theta} = \frac{\beta GM}{(n+1)r_1}\left[-\xi_0\left(\frac{d\theta}{d\xi}\right)_0\right]^{-1}. \tag{31}$$

Hence the approximate proportionality $\bar{\kappa} \propto \theta^{3-n}$ requires

$$-\frac{\sigma-3}{2} = 3-n,$$

or

$$n = \frac{\sigma+3}{2}. \tag{32}$$

13. This determines the polytropic index which the almost complete polytropic distribution must have in order that the equilibrium of Zone C shall be assured. I advance no consideration of plausibility for the assumption that the index n valid in Zone C controls the index value in the interior. Indeed it would be absurd to suppose that it is actually so controlled, and we shall in fact later examine the consequences of avoiding this assumption. At present we are merely examining a tentative hypothesis and its consequences—namely, the hypothesis that the interior constitutes an almost complete polytrope of a single index n—just as Eddington discussed the case $n = 3$, or $\kappa\eta = \text{constant}$.

14. We have now enough relations to substitute for $\bar{\kappa}$ in (24′) and cancel the factor θ^{3-n}. We just use (30) and (31). Regarding β simply as an abbreviation for the right-hand side of (25′), we then get from (24′)

$$\left[\frac{4\pi cGM\kappa_0(\frac{1}{3}a)}{(R/\mu)L}\,\frac{\sigma+5}{8}\right]^{\frac{1}{2}} L = \frac{16\pi^2 cG^4M^3\beta^4(\frac{1}{3}a)}{(R/\mu)^4}\left[\frac{\beta GM}{(n+1)(R/\mu)r_1}\right]^{\frac{\sigma-3}{2}}(n+1)^{-3} \times\left[-\xi_0{}^2\left(\frac{d\theta}{d\xi}\right)_0\right]^{-2}\left[-\xi_0\left(\frac{d\theta}{d\xi}\right)_0\right]^{-\frac{\sigma-3}{2}}. \tag{33}$$

The factors involving ξ_0 and $(d\theta/d\xi)_0$ combine to give

$$\left[-\xi_0{}^{\frac{\sigma+5}{\sigma+1}}\left(\frac{d\theta}{d\xi}\right)_0\right]^{-\frac{\sigma+1}{2}},$$

or, by (32),

$$\left[-\xi_0{}^{\frac{n+1}{n-1}}\left(\frac{d\theta}{d\xi}\right)_0\right]^{-(n-1)}.$$

* In this formula μ is a *mean* through the outer layers, unlike the molecular weight μ in earlier formulæ, which refer to a definite level (in Zone C).

Since θ is here an E-solution, taking the finite value $\theta = 1$ at $\xi = 0$, we have in a standard notation *

$$-\xi_0^{\frac{n+1}{n-1}}\left(\frac{d\theta}{d\xi}\right)_0 = {\omega_n}^0,$$

${\omega_n}^0$ being a constant known numerically † for each index n.

Relation (33) now becomes on squaring

$$\kappa_0 L = \frac{8}{\sigma+5}\,\frac{(4\pi)^3 cG^7M^5(\frac{1}{3}a)\beta^8}{(R/\mu)^7}\left[\frac{\beta GM}{(n+1)(R/\mu)r_1}\right]^{\sigma-3}\frac{(n+1)^{-6}}{[{\omega_n}^0]^{2(n-1)}}. \qquad (34)‡$$

In this relation every constant is determined. The factors β on the right-hand side are not quite constant in Zone C—the polytropic assumption does not quite absorb the variations of $\bar{\kappa}$ in the original equation (24′)—but β is very little different from unity, and can in any case be easily estimated at a representative point in Zone C. Apart from this small mathematical imperfection, relation (34) determines the rate at which the polytrope radiates to space, with proper regard paid to boundary conditions.

For a fixed σ or n, (34) gives the proportionality

$$\kappa_0 L \propto \frac{M^{\sigma+2}\mu^{\sigma+4}\beta^{\sigma+5}}{{r_1}^{\sigma-3}}. \qquad (35)$$

If we use (19) to eliminate r_1, we get

$$\kappa_0 L^{1+\frac{1}{2}(\sigma-3)} \propto M^{\sigma+2}\mu^{\sigma+4}\beta^{\sigma+5}{T_1}^{2(\sigma-3)}. \qquad (36)$$

For example, if $\sigma = 3\frac{1}{2}$, $n = 3\frac{1}{4}$, this reduces to

$$L \propto M^{\frac{22}{5}}\mu^6\beta^{\frac{34}{5}}{T_1}^{\frac{4}{5}}\kappa_0^{-\frac{4}{5}}. \qquad (37)$$

In this proportionality μ is the molecular weight in Zone C, and σ and κ_0 refer to the opacity law in Zone C. The proportionality only holds for an almost complete polytropic interior of fixed index n given by (32). If σ is changed, n changes and the constant of proportionality changes.

15. Proportionality (37) may be compared with the relation obtained by Eddington for the same opacity law, working on the view that the interior opacity is relevant. He took his constant § as given by $n = 3$ instead of our $n = 3\frac{1}{4}$, but his proportionality is

$$L \propto M^{\frac{7}{5}}(1-\beta)^{\frac{3}{2}}\mu^{\frac{4}{5}}{T_1}^{\frac{4}{5}}$$

where

$$1-\beta \propto M^2\mu^4\beta^4,$$

so that his relation gives

$$L \propto M^{\frac{22}{5}}\mu^{\frac{34}{5}}\beta^6{T_1}^{\frac{4}{5}}. \qquad (37′)$$

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* *M.N.*, **92**, 618, 1932.

† In Comrie's *Tables of Emden Functions*, ${\omega_n}^0$ is denoted by Y_n, and is given at the head of each table.

‡ In accordance with a remark in a previous footnote, the factor μ^7 in this formula should strictly be replaced by $\mu^8/\bar{\mu}$, where μ refers to Zone C and $\bar{\mu}$ is a mean value of μ down to Zone C.

§ He corrected for this by an adjustable multiplier α.

Remembering that Eddington had $\kappa_0 = C/\mu$, we see that (37) and (37′) agree *formally* almost exactly; the sole difference is an extra factor $\beta^{\frac{4}{5}}$ in our (37), which arises in connection with the approximations leading to (30), and is in any case immaterial since $\beta \sim 1$.

But the assumptions on which equation (34) or proportionality (37) are based are much less than those made by Eddington, and in (37′) the symbols μ and C refer to the *interior*. Equation (34) holds good on the following assumptions:—

(1) the gas-laws hold in and above Zone C, with definite molecular weight μ;
(2) the law of opacity in Zone C is $\kappa \propto \rho/T^{\sigma}$;
(3) radiative equilibrium holds in and above Zone C;
(4) Zone C and the material interior to it form an almost complete polytrope as regards density-distribution, but the interior may obey any equation of state, law of opacity, mode of transfer of energy or law of generation of energy compatible with this polytropic state.

If, therefore, relation (34) is found to be obeyed by actual stars, *i.e.* by an observed set L, M, T_1, μ, κ_0—and that the associated proportionality (37) is obeyed pretty closely by actual stars we may conclude from Eddington's investigations—all we are entitled to infer is that our assumptions enumerated above hold good. Assumptions (1), (2), (3) relate only to Zone C and the superincumbent material. The only assumption about the interior is assumption (4), and this is only an assumption about the density-distribution. It by no means justifies us in concluding that the interior is in a state of perfect gas, or in radiative equilibrium. It only confirms our assumptions about Zone C.

If indeed we do assume that the interior is wholly gaseous and in radiative equilibrium, with the same opacity law (with the same numerical value for κ_0) holding from Zone C to centre, then it is easy to infer the corresponding source-distribution. It is then readily found that β then slowly increases inwards, and that

$$\eta \propto \beta^{\sigma+4}\frac{n+1-n\beta}{4-3\beta},$$

which is easily shown to give η slowly increasing inwards. But since β departs little from unity in Zone C and closes further on unity as we approach the centre, η is nearly constant, and we have an almost uniform source-distribution. Thus (34) is the solution for a model (standard model) in which, in the usual notation, ϵ is constant. It will be seen that we have found a practically exact solution, with proper attention to "boundary" conditions, for $\epsilon = \text{const.}$, $\kappa = \kappa_0\rho/T^{\sigma}$.

16. The question of interest now is: Have we any independent check on the assumption of the polytropic state $n = \frac{1}{2}(\sigma+3)$ for the interior? The answer is again given by (34). For a given observed set L, M, T_1, μ, κ_0, relation (34) determines the numerical value of $\omega_n{}^0$. If the value so obtained agrees with that given by Emden's or Comrie's tables, then the assumption

is checked. The numerical value of $\omega_n{}^0$ is the only way in which the interior density-distribution manifests itself.

What are we to infer if the numerical value of $\omega_n{}^0$, as determined observationally, disagrees with the tabulated value? Tentatively this would suggest that as we go inwards through and from Zone C the appropriate solution of Emden's equation starts off with the wrong slope. This would mean that it would not correspond to an E-solution with $\theta = 1$, $d\theta/d\xi = 0$ at $\xi = 0$, but be heading either for a central singularity (solution of "centrally condensed" type) or for an intermediate zero of θ (solution of "collapsed" type). In that case the same polytropic conditions cannot persist to the centre. As we have already remarked, this is just what we should expect, for we cannot expect that the interior will be just arranged as that complete polytrope which suits the equilibrium of Zone C. We should in fact expect that *in general* the observationally determined value of $\omega_n{}^0$ will not be the E-value; only as it were accidentally could it come out so. For as we penetrate the interior, energy generation will somewhere set in, and this will change the polytropic index, and therefore the appropriate solution for θ in the earlier layers will not be of E-type. We cannot now use the relations $\rho = \rho_c \theta^n$, $P = P_c \theta^{n+1}$, and y and z do not reduce to unity at the centre. We cannot now solve the differential equation (1) starting from the centre, and we have to revise our work from the start.

This is easily done. We put now instead of (2)

$$r = r_1 x', \qquad \rho = \lambda_1 y', \qquad P = \lambda_2 z'. \tag{38}$$

Then the equation of mechanical equilibrium reduces to

$$\frac{1}{x'^2}\frac{d}{dx'}\left[\frac{x'^2}{y'}\frac{dz'}{dx'}\right] = -y' \tag{39}$$

provided that

$$\lambda_2 = 4\pi G \lambda_1{}^2 r_1{}^2. \tag{40}$$

The value of λ_1 is at our disposal. We then find that instead of (11) we need to employ

$$\frac{y'^2}{z'^3}\left[-x'^2\frac{dz'}{dx'}\right]^2. \tag{11''}$$

In this we can now put,* with a suitable choice of λ_1,

$$y' = \theta^n, \qquad z' = \theta^{n+1}, \qquad x' = (n+1)^{\frac{1}{2}}\xi, \tag{4'}$$

and (39) reduces to Emden's equation. The appropriate solution must have a zero at $x' = x'_0 = 1$, or $\xi_0 = (n+1)^{-\frac{1}{2}}$.

To connect T with θ in Zone C, we proceed as follows. We have

$$\frac{M(r)}{\frac{4}{3}\pi r^3} = \frac{3}{4\pi G}\left[-\frac{1}{\rho r}\frac{dP}{dr}\right] = \frac{3}{4\pi G}\frac{\lambda_2}{\lambda_1 r_1{}^2}\left[-\frac{1}{y'x'}\frac{dz'}{dx'}\right] = -\frac{3\lambda_1}{\xi}\frac{d\theta}{d\xi}, \tag{41}$$

* It was shown in I, p. 525, that according to equation (39) near the zero of y' the variable z' is connected with y' by a relation of polytropic type.

whence, assuming the gas-laws only in Zone C,

$$\frac{R}{\mu}\frac{T}{\theta}=\frac{p}{\rho\theta}=\frac{\beta\lambda_2}{\lambda_1}=\beta 4\pi G\lambda_1 r_1^{\,2}=\frac{\beta GM}{r_1}\left[-\frac{1}{\xi_0}\left(\frac{d\theta}{d\xi}\right)_0\right]^{-1}=\frac{\beta GM}{(n+1)r_1}\left[-\xi_0\left(\frac{d\theta}{d\xi}\right)_0\right]^{-1}, \quad (42)$$

which is formally the same as (31). Operating on (24′) in the same way as we derived (34) we now get $n=\frac{1}{2}(\sigma+3)$ and

$$\kappa_0 L=\frac{8}{\sigma+5}\frac{(4\pi)^3 cG^7M^5(\frac{1}{3}a)\beta^8}{(R/\mu)^7}\left[\frac{\beta GM}{(n+1)(R/\mu)r_1}\right]^{\sigma-3}\frac{(n+1)^{-6}}{\omega_n{}^{2(n-1)}} \quad (34')$$

with

$$\omega_n=-\xi_0^{\frac{n+1}{n-1}}\left(\frac{d\theta}{d\xi}\right)_0.$$

This is formally the same as (34), with ω_n replacing * $\omega_n{}^0$. Since ω_n is well known to be invariant under different normalisations of θ to different zeros ξ_0, we need no longer pay attention to our particular normalisation of θ to the zero $\xi_0=(n+1)^{-\frac{1}{2}}$; ξ_0 and $(d\theta/d\xi)_0$ are involved only in the combination ω_n.

An observed set L, M, T_1, μ, κ_0 now determines the value of ω_n which *selects* that solution of Emden's equation which is the starting-off solution for the interior. In general, this solution will not persist far beyond Zone C. For it would be physically absurd to suppose that nature suits our mathematical convenience—just as it would be physically absurd to suppose that nature arranges the interior generation of energy so that $\kappa\eta=$ const. The assumptions that $\kappa\eta=$ const., that the interior is a complete polytrope of index $n=\frac{1}{2}(\sigma+3)$, or that the ascertained value of ω_n determines the whole interior, are on the same footing. For example, any substantial increase of ϵ inwards will cause a departure from the solution, with a consequent change in the interior density-distribution. This will be equivalent to some kind of cored configuration. And even if ϵ behaves in an appropriate manner, if $\omega_n\neq\omega_n{}^0$ we must get eventually a cored configuration. From this point onwards the fit of the exterior layers on to the core follows the lines of my papers on the analysis of stellar structure of 1930 and 1932, which worked out the circumstances of fit in particular cases.

The point of interest here is that we can really tell now from observation, supposing sufficient accuracy in the values of L, M, T_1, μ, κ_0, whether we have to do with an almost complete polytrope $n=\frac{1}{2}(\sigma+3)$ or a cored configuration starting off with a singularity-possessing solution of Emden's equation. In either case we have a definite piece of information about the internal density-distribution. If the value of ω_n equals $\omega_n{}^0$, the polytrope is almost complete, or starts off as if it were going to be, and we then know the value of $\bar{\rho}/\rho_c$ from Emden's tables. If $\omega_n<\omega_n{}^0$, we tend to get a configuration of relatively high central density; if $\omega_n>\omega_n{}^0$, we tend to get a relatively more homogeneous density-distribution, of the kind found in

* $\omega_n{}^0$ is the particular numerical value of ω_n for a complete polytrope extending to the centre without central singularity. Solutions of Emden's equation which do not reach the centre without central singularity, or without arriving at an intermediate zero, start from a zero of θ with a value of ω_n unequal to $\omega_n{}^0$.

white dwarfs.* Thus, as I originally inferred in these studies, the single piece of information we can get from an observed set L, M, T_1, μ, κ_0, in the absence of a knowledge of the distribution of η, is something of the general nature of the internal density-distribution.

17. *Physical Explanation.*—We have now to consider physically how it comes about that the exterior opacity is alone relevant in the (M, L, T_1) relation, and why this relation only involves the gas-laws in the exterior layers. It must not be thought that the exterior opacity determines the luminosity of the star. Our analysis would apply equally well if the star had a radio-active core generating energy at a rate independent of physical conditions. It applies also in any other circumstances, whatever the physical law of generation of energy. The star must always adjust itself so that the net amount of energy radiated per second through Zone C equals the amount of energy generated internally per second. For Zone C to be in this condition the exterior mean opacity in Zone C must behave in a certain way; both its absolute values and its behaviour with depth must fit. These absolute values depend on the values of T in Zone C, which in turn depend on the radius of the star. The star thus carries out the adjustment by altering its radius.† When this adjustment is carried out (34) is just satisfied, or (36) with the proper constant of proportionality. This condition involves κ_0, the coefficient in the opacity law. The condition expresses the thermal and mechanical equilibrium of the exterior layers.

Suppose we construct a model configuration in *mechanical* equilibrium of definite mass and radius. It will be generating energy at some definite rate E, and the outer layers will be radiating to space at some definite rate L. The star has to adjust itself so that $L=E$, which it must do by adjusting its radius. In the course of this adjustment L certainly changes; E may or may not change. An adjustment of the exterior layers alone will probably affect E very little, but it will alter the radius, and so L, until L agrees with E. Clearly the condition $L=E$ is a necessary but not a sufficient condition for a thermally steady state. There will be a condition $L(r)=E(r)$ which must be satisfied at every level r. This condition is readily expressed in a form comparable with the condition employed in the main analysis of this paper. We readily prove in fact that at any level r the corresponding value of β satisfies both the relations

$$1-\beta=\frac{\overline{\kappa\eta}L}{4\pi cGM}, \tag{43}$$

where

$$\overline{\kappa\eta}=\frac{1}{P-P_0}\int_{P_0}^{P}\kappa\eta dP, \tag{44}$$

* The mathematical theorems here needed are contained in R. H. Fowler's paper, *Quart. Journ. Math.* (*Oxford*), **2**, 259, 1931. See his Theorem IX, p. 276, and more especially his Tables I, II, pp. 281, 284, Case IV, with "σ" $=-4$, yielding $3<n<5$. (Fowler's "σ" has a different meaning from our σ.)

† The argument assumes $\sigma>3$. If $\sigma=3$, the adjustment of radius alone has no influence on the opacity, and the ultimate adjustment would come through the effect of change of radius on energy-generation, as explained in § 19, Paper I.

and
$$\frac{\beta^4}{1-\beta}=\frac{(R/\mu)^4}{4\pi G^3[M(r)]^2(\frac{1}{3}a)}\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2, \tag{45}$$

provided the material at level r is gaseous. Of these the first, (43), expresses that the opacity exterior to r just permits the outflow of the radiation $E(r)$ generated inside r; and the second, (45), which follows as usual from the continuity of p, ensures the mechanical equilibrium of the matter interior to r. The parameter β may vary in any manner through the star. But we cannot know the value of $\overline{\kappa\eta}$ at levels interior to Zone C in the absence of a knowledge of the mode of energy-generation. Thus these relations, whilst satisfied at every level, are useless save in Zone C, where $\eta=1$, $M(r)\sim M$, $x\sim x_0$.

For example, if we try to apply (43) and (45) to the centre, we get from (43),
$$1-\beta_c=\frac{(\overline{\kappa\eta})_c L}{4\pi cGM},$$
but (45) becomes useless for the determination of β_c. Thus these equations are of no avail for a calculation of L.

We see that the (L, M, T_1) relation is just one of the conditions of equilibrium, namely, the condition containing the "global" parameters. This condition contains the numerical coefficient ω_n, where $n=\frac{1}{2}(\sigma+3)$. If the star is assumed to be an almost complete polytrope of index n, then the value of ω_n is known, namely, $\omega_n{}^0$; otherwise it is unknown. A complete knowledge of the interior would provide the value of ω_n, namely, the value necessary for the fit of Zone C on to the interior. Presumably different types of interiors will be characterised by different values of ω_n. It would be interesting to compare giants and dwarfs from this point of view.*

When the star is adjusted to equilibrium, the radius suits the value of κ_0, for $\sigma>3$. This determines the mean density. If the star is a complete polytrope its central density is known, and so the general march of the density-distribution is known. To illustrate other cases, consider what would happen if κ_0 were artificially changed. Then by (34′), if E, and so L, is unaltered, either r_1 or ω_n or both must change. A *homologous* expansion, leaving the star a polytrope of the same index, and therefore ω_n unaltered, would change r_1 alone. But it is difficult to think that a change of κ_0 would affect the deeper layers. Consequently the expansion would be unlikely to be homologous. Consequently ω_n must change. Physically this means that the expansion or contraction will affect chiefly the material in the neighbourhood of Zone C; an expansion, for instance, will reduce $\bar{\rho}$ whilst probably leaving ρ_c unaltered. We have then an extended outer zone. For example, decrease of κ_0 without change of L means increase of $r_1{}^{\sigma-3}\omega_n{}^{2(n-1)}$. If ω_n is unaltered r_1 is increased, and the outer layers expand. If ω_n diminishes, there will be an increased expansion. Diminution

* Effects attributed in current investigations to differing hydrogen contents would on the present analysis be attributed to differing values of ω_n, *i.e.* to differing types of internal density-distribution.

of ω_n means a tendency towards a central condensation ; physically, compared with the complete polytrope, a relatively greater fraction of the mass is concentrated towards the centre and a relatively smaller fraction in the outer layers, which is consistent with the inferred expansion of the outer layers. If, on the other hand, ω_n increases, the tendency of r_1 to expand will be diminished, and there will be a tendency towards a "collapsed" solution ; compared with the complete polytrope, the configuration will be more homogeneous, and an increased fraction of the mass will be in the outer layers. In both cases the absolute value of ρ_c may be unaltered, but $\bar{\rho}/\rho_c$ will be more decreased in the first case than in the second case. All that can be inferred about the internal density-distribution of an observed star, in the absence of a specific theory of energy-generation, is the value of ω_n and its consequences.

None of this conflicts with Vogt's theorem, to the effect that when the mode of energy-generation is given L is a function of M only. The combination of Vogt's relation $L=F(M)$ with the equilibrium relation $L=\phi(M, T_1)$ simply determines the effective temperature T_1 as a function of M. The observed distribution of T_1 with M (or L) (the Russell diagram phenomenon) must therefore involve the physics of energy-generation.

18. *Connection with the Interior.*—It seems worth while to show exactly where in our formulæ the linkage occurs with the interior, for formulæ of this type have been criticised in the past as being purely photospheric formulæ. That the criticism is ill-founded is seen from the particular case where the interior density-distribution is that of a polytrope of index $n=3$, when the numerical coefficient (11) has the value $(3+1)^3(2{\cdot}018\ldots)^2$, a value derived by integration of Emden's equation of index 3 from the centre outwards ; the linkage with the interior is here as explicit as possible.

But consider the coefficient (11) more closely. If we change back from the variables y, z, x to the variables ρ, P, r by means of (2), we find

$$\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2=4\pi G\frac{\rho^2}{P^3}\left[-r^2\frac{dP}{dr}\right]^2,$$

or, since in this neighbourhood

$$\frac{dP}{dr}=-\frac{GM}{r^2}\rho,$$

we have in Zone C

$$\frac{y^2}{z^3}\left[-x^2\frac{dz}{dx}\right]^2=4\pi G^3M^2\frac{\rho^4}{P^3}. \tag{46}$$

If we insert this expression in (24′) the resulting relation involves, of course, only variables relevant to Zone C, and contains no reference to the interior. In returning to the variables ρ, P we reverse our algebra, and *lose the connection with the interior.* The connection with the interior is contained in equation (46), in which the variables on the right-hand side refer only to Zone C, and the value of the left-hand side depends on, and can be calculated from, the march of $z=f(y)$ in the interior, by integration of (1) from the

centre outwards with the initial conditions (3). The numerical value of the left-hand side of (46) at any level in Zone C, *i.e.* the value of ρ^4/P^3, is fixed by the assumptions for the interior which determine $z=f(y)$. To use (46) in (24′) or (27) is to lose the connection with the interior.

In fact, if we insert (46) on the right-hand side of (27) we get simply the identity

$$\frac{\beta^4}{1-\beta}=\frac{(R/\mu)^4}{\frac{1}{3}a}\frac{\rho^4}{P^3},$$

which follows from the definition of β and the gas laws. The connection with the interior disappears altogether. Equation (24′) is equivalent to (26′) and (27) combined. To maintain that (24′) contains no reference to the interior would be equivalent to maintaining that (27) contains no reference to the interior, *i.e.* that the famous quartic equation contains no reference to the interior.

19. *Summary.*—The paper traces the fit of the exterior layers of a star with the interior by following the continuity of gas-pressure p. These exterior layers are shown to be divisible into three zones, of which the third, Zone C, is that region in which the effect of non-zero boundary temperature has disappeared, but which is the "effective boundary" from the point of view of the equilibrium of the interior. The result of equating the value of p found by working from the boundary inwards to Zone C, to the value of p found by working from the centre of the star outwards to Zone C, is a relation between luminosity L, mass M, molecular weight μ in Zone C and mean exterior opacity $\bar{\kappa}$ down to Zone C, involving a dimensionless coefficient depending on the internal density-distribution. From this it is shown that Eddington's quartic equation and luminosity formula determine the mean exterior opacity in Zone C (and not the interior opacity) when the interior is a polytrope of index $n=3$. It is concluded that the inferences usually made as to the gaseous character of the interior and its hydrogen content really apply only to Zone C.

Following a suggestion due to Russell, the analysis is extended to cover any law of exterior opacity of the type $\kappa=\kappa_0\rho/T^\sigma$ ($\sigma>3$). It is shown that if the interior is a *complete* polytrope compatible with the equilibrium of Zone C, then its index n is $\frac{1}{2}(\sigma+3)$. The above-mentioned relation then reduces to one between L, M, T_1, μ, κ_0, involving $\omega_n{}^0$, the Emden invariant $-\xi_0^{\frac{n+1}{n-1}}(d\theta/d\xi)_0$, given numerically in the standard tables. For $\sigma=3\frac{1}{2}$, the relation practically coincides with Eddington's semi-empirical formula, save that the new rigorous form contains the values of μ and κ_0 appropriate to the exterior layers, and that no assumptions are now made about the equation of state or opacity in the interior.

For an observed star for which L, M, T_1, μ, κ_0 are accurately known, we can in principle calculate the value of the coefficient $\omega_n{}^0$ from observation, and compare it with the theoretical tabulated value. We can thus test the assumption that the interior is a complete polytrope.

If the value of $\omega_n{}^0$ thus inferred disagrees with the tabulated value, the situation has to be reconsidered. Further analysis shows that the same

(L, M, T_1, μ, κ_0) relation formally holds with ω_n replacing $\omega_n{}^0$, where ω_n is the same invariant evaluated for a *non-Emden* solution θ of the polytropic equation. An observed set L, M, T_1, μ, κ_0 then determines the numerical value of ω_n appropriate to the star. As above, if $\omega_n = \omega_n{}^0$, the star behaves as if its interior were distributed according to a complete polytrope; if $\omega_n < \omega_n{}^0$, the interior is relatively more "centrally condensed" and the outer layers relatively more extended; if $\omega_n > \omega_n{}^0$, the interior is relatively more homogeneous and the outer layers relatively less extended. It is thus possible in principle to determine from external observation something of the nature of its internal density-distribution.

It is probable that differences between stars of different categories, previously attributed to differences of hydrogen content, should be attributed to differences in the numerical values of ω_n and so to differences in their internal density-distributions.

It is shown that a star must adjust its radius to suit the value of κ_0; Zone C can in this way always adjust its opacity to suit any interior circumstances, but in so doing it adjusts its extension, and in turn the radius and mean density of the star. The exact nature of the linkage between the equilibrium of Zone C and that of the interior is analysed.

It is a pleasure to express my indebtedness to Professor H. N. Russell, to whose stimulus the paper owes its inception. I wish also to thank Professor H. H. Plaskett for his helpful discussion of many points.

Mathematical Appendix

Note 1. *A formula for* $\bar{\kappa}$.—If p' is the radiation-pressure, from the equations

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho, \qquad \frac{dp'}{dr} = -\frac{\kappa\rho}{c}\frac{L(r)}{4\pi r^2}, \tag{47}$$

we have in Zone C

$$\kappa dP = \frac{4\pi cGM}{L}dp'.$$

Hence, in any interval of P,

$$\bar{\kappa} = \frac{\int \kappa dP}{\int dP} = \frac{\int dp'}{\int \frac{dp'}{\kappa}},$$

or

$$\frac{1}{\bar{\kappa}} = \frac{\int \frac{1}{\kappa} dp'}{\int dp'}. \tag{48}$$

*Note 2. Evaluation of $\bar{\kappa}$ in Zone C.**—From equations (47), with $\kappa = \kappa_0\rho/T^\sigma$, we have

$$1 + \frac{dp}{dp'} = \frac{4\pi cGM}{\kappa_0 L}\frac{T^\sigma}{\rho} = \frac{4\pi cGM(R/\mu)}{\kappa_0 L}\frac{1}{p}\left(\frac{p'}{\frac{1}{3}a}\right)^{\frac{\sigma+1}{4}}.$$

If $p'^{\frac{\sigma+1}{4}}/p$ increases with T, the right-hand side of this becomes ultimately large compared with unity. Hence, under these conditions we have in Zone C approximately

$$\frac{dp}{dp'} = \frac{4\pi cGM(R/\mu)}{\kappa_0 L}\frac{1}{p}\left(\frac{p'}{\frac{1}{3}a}\right)^{\frac{\sigma+1}{4}}.$$

Integrating, we can in Zone C take the constant of integration as zero, and we then have

$$p = \left[\frac{4\pi cGM(R/\mu)(\frac{1}{3}a)}{\kappa_0 L}\frac{8}{\sigma+5}\right]^{\frac{1}{2}}\left(\frac{p'}{\frac{1}{3}a}\right)^{\frac{\sigma+5}{8}},$$

which will be a valid approximation provided

$$\frac{\sigma+1}{4} - \frac{\sigma+5}{8} > 0,$$

or

$$\sigma > 3.$$

With this approximation,

$$\kappa = \frac{\kappa_0}{R/\mu}\frac{p}{T^{\sigma+1}} = \left[\frac{4\pi cGM\kappa_0(\frac{1}{3}a)}{(R/\mu)L}\frac{8}{\sigma+5}\frac{1}{T^{\sigma-3}}\right]^{\frac{1}{2}}. \tag{49}$$

Applying the result of Note 1, we find readily

$$\frac{1}{\bar{\kappa}} \sim \frac{8}{\sigma+5}\frac{1}{\kappa}, \tag{50}$$

whence in Zone C

$$\bar{\kappa} = \left[\frac{4\pi cGM\kappa_0(\frac{1}{3}a)}{(R/\mu)L}\frac{\sigma+5}{8}\frac{1}{T^{\sigma-3}}\right]^{\frac{1}{2}}. \tag{51}$$

This is formula (30) of the main paper.

Note 3. Evaluation of T/θ.—If $P = P_c\theta^{n+1}$, $\rho = \rho_c\theta^n$, *without using the gas-laws in the interior* we have at any level where the material is gaseous

$$\frac{R}{\mu}\frac{T}{\theta} = \frac{p}{\rho\theta} = \frac{\beta P}{\rho\theta} = \frac{\beta P_c}{\rho_c}.$$

Now from (2),

$$r_1 = \left(\frac{P_c}{4\pi G\rho_c^2}\right)^{\frac{1}{2}}x_0 = \left(\frac{P_c}{4\pi G\rho_c^2}\right)^{\frac{1}{2}}(n+1)^{\frac{1}{2}}\xi_0,$$

and

$$\frac{\bar{\rho}(r_1)}{\rho_c} = -\frac{3}{\xi_0}\left(\frac{d\theta}{d\xi}\right)_0.$$

* The essence of this calculation is well known. I give it for the sake of completeness.

Using these to eliminate P_c and ρ_c, we have

$$\frac{R}{\mu}\frac{T}{\theta}=\beta\frac{4\pi G{r_1}^2}{(n+1){\xi_0}^2}\frac{M}{\frac{4}{3}\pi {r_1}^3}\left[-\frac{3}{\xi_0}\left(\frac{d\theta}{d\xi}\right)_0\right]^{-1}=\frac{\beta GM}{(n+1)r_1}\left[-\xi_0\left(\frac{d\theta}{d\xi}\right)_0\right]^{-1}. \quad (52)$$

This is formula (31) of the main paper. The result is well known when the whole interior is gaseous, but it does not appear to have been derived before under the present less stringent conditions.

Note 4. *Behaviour of* η *in the Polytrope* $n=\frac{1}{2}(\sigma+3)$.—Assuming radiative equilibrium and the gas-laws, we have

$$1-\beta=\frac{p'}{P}\propto\frac{T^4}{P}\propto\frac{(\beta\theta)^4}{\theta^{n+1}}\propto\beta^4\theta^{-(n-3)}. \quad (53)$$

Hence $(1-\beta)/\beta^4$ decreases as θ increases. Hence β increases inwards. Moreover, by logarithmic differentiation we find

$$\frac{\theta}{\beta}\frac{d\beta}{d\theta}=\frac{(n-3)(1-\beta)}{4-3\beta}. \quad (54)$$

Now, since

$$\kappa\eta\propto\frac{dp'}{dP},$$

we have

$$\eta\propto\frac{T^\sigma}{\rho}\frac{d(T^4)}{dP}$$

$$\propto\frac{(\beta\theta)^\sigma}{\theta^n}\frac{d[(\beta\theta)^4]}{d\theta^{n+1}}$$

$$\propto\beta^{\sigma+4}\theta^{\sigma+3-2n}\left[1+\frac{\theta}{\beta}\frac{d\beta}{d\theta}\right].$$

But $n=\frac{1}{2}(\sigma+3)$. Hence, using (54),

$$\eta\propto\beta^{\sigma+4}\frac{n+1-n\beta}{4-3\beta}. \quad (55)$$

This is the result quoted towards the end of § 15 of the main paper.

We see that since β increases inwards, η will increase inwards provided

$$\frac{\sigma+4}{\beta}-\frac{n}{n+1-n\beta}+\frac{3}{4-3\beta}>0,$$

i.e. since $n=\frac{1}{2}(\sigma+3)$, provided

$$\tfrac{1}{2}(\sigma-3)\beta<(\sigma+4)(4-3\beta)[\tfrac{1}{2}(\sigma+5)-\tfrac{1}{2}(\sigma+3)\beta].$$

For $\beta=1$, the left-hand side is $\frac{1}{2}(\sigma-3)$ and the right-hand side is $\sigma+4$, and so the inequality is satisfied. For smaller values of β, the left-hand side is smaller and the right-hand side is bigger, and the inequality is more pronounced. Hence η increases inwards. Since β is in Zone C nearly unity and approaches unity as we go inwards, η is approximately constant, and the source-distribution is nearly uniform.

University Observatory, Oxford:
1937 *October.*

OBJECTIVE PRISM OBSERVATIONS OF NOVA LACERTÆ (1936).

D. L. Edwards and D. R. Barber.

The main object of this paper is to give a general descriptive account of the spectrum of Nova Lacertæ in its various stages up to 1936 September 3, as observed with an objective prism at the Norman Lockyer Observatory. The absence of any method of standardising the plates has precluded any detailed discussion of line intensities, but rough visual estimates have been noted, partly as an indication of the presence or absence of particular lines on any date, and partly to show their relative prominence in the spectrum. In this type of instrument no comparison spectrum can be impressed on the plate, so that accurate determinations of wave-lengths and radial velocities are not possible. The presence, however, of strong interstellar lines of [*H*] and [*K*] in some of the spectra made it possible to obtain reasonably good approximate values, and in view of the exceptionally large velocities involved (amounting to nearly 4000 km./sec. in some cases) it was considered worth while to include a short discussion of results obtained in this way.

The instrument used was the 12-inch McClean telescope with a 20° prism, giving a dispersion ranging from 48 A./mm. at $H\beta$ to 23 A./mm. at $H\epsilon$. The plates used in all cases were Barnet "Super Press."

Description of the Sequence of Spectral Changes

The first photograph secured, on the morning of June 20, showed the spectrum to be predominantly absorption and typical of α Cygni, with the exception that all the characteristic lines were displaced to the violet and broadened. As a result many close and narrow lines were blended. Strong and sharp *Ca*II interstellar lines were prominent against a strong continuous background. No emission lines were detected with certainty, although $H\beta$ may have been present. By June 22, however, the emission spectrum was well developed, strong bands of *H*, *Ca*II, *Fe*II, *Ti*II, *Cr*II, *Sc*II and *Sr*II being present. The violet displacement of the absorption spectrum had increased, and in addition two new components of the Balmer lines appeared, one of higher and one of lower velocity than the first. The low velocity component did not persist after June 24. The interstellar [*H*] and [*K*] lines were still very prominent. Plates secured on the evening of June 22 show slight differences from those exposed on the morning of the same day, chiefly in the increased strength and displacement of the Balmer absorption lines, and an increased diffuseness of the metallic absorptions causing still further blending. A second absorption component of *Ca*II

appeared on this date corresponding in displacement to the high velocity Balmer components.

During the period June 23–24 the spectrum was more complex. The emission lines exhibited increased strength and showed a tendency to broaden. The continuous background was commencing to fade, and this, combined with a broadening of the absorption lines, tended to make the latter less conspicuous. The second components of the Balmer series and of *Ca*II were, however, strong and moderately sharp. By June 23 the *Mg*II absorption at 4481 had almost disappeared, having been of moderate strength in the earlier spectra. On this date also the Balmer emission lines began to show a more complex structure. Features which might have been absorption lines, or merely gaps in the structure of the emission bands, appeared nearly centrally on the emission, and no satisfactory metallic absorptions could be found as origins. In the case of $H\beta$ this central "absorption" lay between the displaced positions of 4884·6 (*Cr*II) and 4900·0 (*Ba*II), but was not close enough to either of these to be assigned a single origin, nor wide enough to be a blend of both. Subsequent slight changes of position (mostly to the red side of the centre) occurred simultaneously in $H\beta$ and $H\gamma$, so that there is little doubt that they represented structural features of the Balmer emission bands rather than simple absorption lines. In the case of $H\delta$, however, this feature must have blended with the displaced *Si*II lines (4128·0, 4130·9) which were prominent on the previous day. The interstellar [*H*] and [*K*] lines were now becoming faint.

The first indication of a change in spectral type from A to B occurred on June 24, when numerous absorption lines appeared, attributed to *N*II and *O*II, some of which were faint and sharp. In addition, the *He*I lines at 4026·2, 4413·8, 4471·5 were faintly visible in absorption.

The next phase of development extended throughout the period June 29–July 11, and was marked by the gradual disappearance of the majority of absorption and metallic emission lines, yielding a typical "4640" emission spectrum in which bands due to *H* and *N* were alone prominent. At the commencement of this period the rate of increase in the Doppler displacement of the remaining absorption lines gradually diminished, and by July 7 the displacement had become almost stationary. On this date the two remaining Balmer components gave approximate radial velocities of −2000 and −3500 km./sec. respectively. By July 8 these had faded out. Throughout this phase absorption lines (strongly displaced) due to *N*III made sporadic appearances. On July 3 and 5 a blend of the doublet 4097·3, 4103·4—probably also blended with $H\delta$ absorption—was possibly present. On July 7 these lines were very conspicuous and were accompanied by 4335·5, 4339·5, 4348·4, the latter showing as two well-separated and fairly sharp lines of moderate strength. It is probable that the lines 4335·5 and 4339·5 were blended. The lines 4634·2, 4640·6 were also present, possibly being blended with *N*II (4630·6, 4643·1). All the *N*III lines on this date showed a displacement of approximately −2600 km./sec. (derived from rough measures). On July 8, 10, 11 and 16 some of these lines (particularly 4097·3, 4103·4) were definitely present in moderate strength, but they were

not seen on any later date. The Balmer emission bands showed a complex and varying structure during this period. By July 9 the central absorptions had disappeared, and on July 3 a strong maximum near the red edge of each of the emission bands was evident. This became more sharply defined by July 5, and afterwards continued to increase in strength, at the same time becoming wider and more diffuse. The two prominent bands at 4641 and 4534, which had been features of the earlier emission spectrum, suddenly widened on July 3, presumably due to the appearance of numerous *N*II, *N*III, *C*III and *Si*III lines.

The last phase to be described covers the period July 11–September 3, and is characterised by the appearance of the typical nebular spectrum. "Forbidden" lines of [*O*III] first appeared faintly in emission at 4958·9 and 5006·8 on July 11, and these increased gradually in strength until, on September 3, they were conspicuous features of the spectrum. The [*O*III] line at 4363·2 was possibly also present on the former date, but the first definite indication appeared on July 16. The proximity of the broad $H\gamma$ emission band made identification difficult. This line was moderately strong on August 1, and continued to strengthen until the last observation on September 3. On July 24, bands due to *O*II, *N*II, *N*III and *He*II were present in moderate strength at 4190, 4240 and 4686, whilst fainter bands were noted at 4379 and 4471. The band at 4641 remained very conspicuous throughout the period, but the blend of *N*II and *N*III lines at 4533 diminished in strength after July 11 and had faded out by July 25.

On July 16 two fairly sharp and moderately strong absorption lines were noted at approximately 4562, having a separation of 14·5 A. Their origin is attributed to *Si*III 4552·6, 4567·8 ; but, assuming the identification to be the correct one, the displacement would indicate a velocity of only −1000 km./sec. (approximately).

During this period another feature of interest was the marked variability in the structure of the Balmer emission bands. By July 20 the pronounced maximum on the red edge of each band had become very conspicuous. Two maxima appeared in $H\gamma$ on August 1 in place of the previous single maximum. Owing to the smaller dispersion at $H\beta$ the separation of the two maxima was not well marked although doubtless present. On August 11 the structure of $H\gamma$ showed little change, although no maxima could be seen with certainty in the $H\beta$ band. The structure of the $H\gamma$ band subsequently underwent a gradual change until, by August 31, the maximum had become diffuse and there was no sign of the previous splitting. The single diffuse maximum showed, however, a slight displacement from its former position towards the violet. On this latter date the only additional prominent emission bands were 5007, 4959 ([*O*III]), 4641 (*N*II) and 4363 ([*O*III]).

Table I. gives the absorption lines which appeared during later stages of development of the nova. Notes are included regarding the dates on which the lines appeared, and their approximate displacements (in km./sec.). The displacements could be determined only very roughly on account of the absence of measurable interstellar lines. Approximate wave-lengths and displacements were obtained by relative measures from hydrogen emission

lines. They are subject to correction for the velocity displacements of the latter.

TABLE I

Absorption Lines in Later Negatives

Observed λ	Probable Origin		First Appearance	Last Appearance	Approximate Displacement
4001	4026·2	*He*I	June 24	June 24	−1900 km./sec.
4065	4097·3	*N*III	July 3	July 16	−2600 ,,
	4103·4	*N*III			
4118	4143·8	*He*I	June 24	June 24	−1900 ,,
4300	4335·5	*N*III	July 7	July 7	−2600 ,,
	4339·5	*N*III			
4311	4348·4	*N*III	,, 7	,, 7	−2600 ,,
4443	4471·5	*He*I	June 24	June 24	−1900 ,,
4474	4510·9	*N*III	July 7	July 7	−2600 ,,
	4514·9	*N*III			
4537	4552·6	*Si*III	,, 16	,, 16	−1600 ,,
4553	4567·8	*Si*III	,, 16	,, 16	−1000 ,,
4569	4591·0	*O*II	June 24	,, 7	−1900 ,,
	4596·2	*O*II			
	4601·5	*N*II			
4601	4630·6	*N*II	,, 24	*	−1900 ,,
4594	4634·2	*N*III	July 3	July 16	−2600 ,,
4610	4638·9	*O*II	June 24	*	−1900 ,,
4600	4640·6	*N*III	July 3	July 16	−2600 ,,
4612	4641·8	*O*II	June 24	*	−1900 ,,
4614	4643·1	*N*II	,, 24	*	−1900 ,,
4620	4649·1	*O*II	,, 24	June 29	−1900 ,,
4621	4650·9	*O*II	,, 24	,, 29	−1900 ,,
4647	4676·2	*O*II	,, 24	,, 29	−1900 ,,

* Possibly present on later dates, but blending with *N*III or masked by emission band.

Table II shows the principal emission bands which were measured in the earlier negatives (up to June 29), with their mean widths in angstroms at two different periods. Probable origins are suggested in cases where the emissions are comparatively narrow, but in the case of the broader bands the number of possible contributory sources is so great that no useful purpose would be served by enumerating them all. No mention is made of *H* or *Ca*II, but of course the Balmer lines and [*H*] and [*K*] persist with more or less complexity of structure throughout the whole series.

In Table III the emission bands which appeared later in the life of the nova are given, with the dates at which they first appeared. During this stage the majority of the bands in Table II had weakened or faded out completely.

Emission-band widths were measured whenever practicable in negatives covering the period June 22–July 25, and some of the results are depicted in

TABLE II

Principal Emission Bands in Early Spectra

(R = see Remarks)

Probable Origin		Measured Centre	Mean Width June 22–23	Mean Width June 24–29	Remarks
R		4175	40 A.	33 A.	Blend of numerous metallic sources.
4215·5	*Sr*II	4216	R	...	Strong, fairly sharp.
R		4240	51	40	Blend of numerous lines.
4246·8	*Sc*II	4247	R	...	Mod. strong. Sharp.
R		4298	26	18	Blend of numerous lines.
R		4405	19	...	,, ,, ,,
4450·5	*Ti*II	4450	R	R	Narrow. Faint.
4501·3	*Ti*II	4505	28	19	Possibly includes 4493·5 (*Ti*II) and 4515·3 (*Fe*II).
4507·2	*Cr*II				
4508·3	*Fe*II				
4529·5	*Ti*II	4535	...	17	
4534·0	*Ti*II				
4541·5	*Fe*II				
R		4575	38	21	Blend of numerous lines.
4629·3	*Fe*II	4641	25	17	
4634·1	*Cr*II				
4635·3	*Fe*II				
4657·2	*Ti*II				
4713·1	*He*	4714	...	R	Mod. sharp. 4715·1 *Cr*II ?
4715·1	*Cr*II	4718	18	...	
4719·5	*Ti*II				
4923·9	*Fe*II	4924	44	47	
5018·5	*Fe*II	5019	54	50	

TABLE III

Later Emission Bands

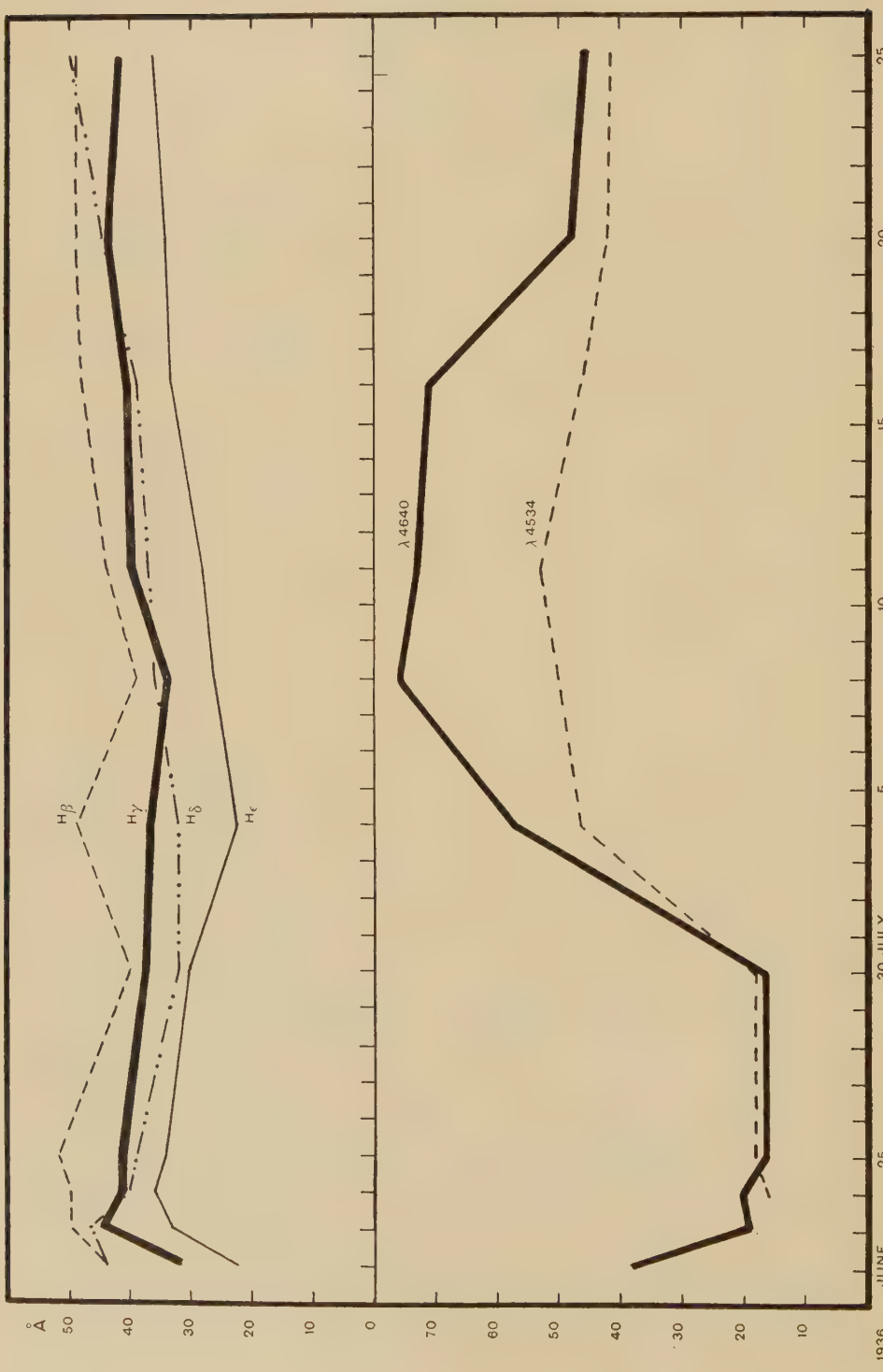
Probable Origin		Approx. Centre	First Appearance	Remarks
4072·2	*O*II	4087	July 5	Blends with *Hδ*, extending down to λ 4070.
4075·9	*O*II			
4088·9	*Si*IV			
4116·1	*Si*IV			
4097·3	*N*III			
4103·4	*N*III			
4185·5	*O*II	4190	,, 11	Faint broad band, fading out after July 20.
4187·1	*C*III			
4189·8	*O*II			
4195·7	*N*III			
4200·0	*N*III			
4199·9	*He*II			

TABLE III—*continued*

Probable Origin		Approx. Centre	First Appearance	Remarks
4237·0	*N*II	4240	July 11	Faded out after July 26.
4241·8	*N*II			
4363·2	[*O*III]	4363	,, 16*	Continued and strengthened.
4379·1	*N*III	4379	,, 23	Faint. Faded out after July 25.
4471·5	*He*I	4471	,, 16	Faded out after August 1.
4510·9	*N*III	4533	,, 3	Previous band at 4535 commenced to widen and strengthen. Faded out after July 24.
4514·9	*N*III			
4530·4	*N*II			
4541·6	*He*II			
4552·6	*Si*III			
4630·6	*N*II	4641	,, 3	Approximate date when previous band at 4641 commenced to widen.
4634·2	*N*III			
4640·6	*N*III			
4643·1	*N*II			
4647·4	*C*III			
4685·8	*He*II	4686	,, 24	Continued till last observation.
4958·9	[*O*III]	4959	,, 11	Faint, becoming stronger.
5006·8	[*O*III]	5003	,, 11	Becoming stronger.
5001·5	*N*II			
5005·1	*N*II			

* Possibly present on July 11.

the figure (p. 48). The dependence of the Doppler widening upon wavelength of the lines is well shown in the case of the Balmer lines $H\beta$ to $H\delta$ in the upper diagram. Mean measures of line widths (in angstroms) over the period June 22–July 8 gave values of 46, 42 and 38 A. for $H\beta$, $H\gamma$ and $H\delta$ respectively. These agree consistently with the theoretical values computed on the assumption of strict proportionality between line width and wavelength, viz. 46, 41 and 39 A. The observed mean width for $H\epsilon$ is, however, smaller than the computed value by 9 A., but this is probably accounted for by a photographic effect resulting in a general loss of density in this region of the spectrum. It will be seen from the diagram that some overlapping exists, particularly for $H\delta$, and a possible explanation may be the masking effect of adjacent absorption. The comparatively sudden transition from the α Cygni to the "4640" stage is strikingly shown in the lower diagram. In this, line widths have been plotted against date of observation for the bands 4640 and 4535. During the α Cygni stage these latter consisted of blended *Cr*II, *Ti*II and *Fe*II lines, the mean width being approximately 20 A. With the appearance of the *N*III and *N*II lines in emission the width suddenly increased, and rapidly attained a maximum of 75 A. in the case of 4640 on July 8, and of 53 A. for 4534 on July 11. In later negatives the bands became narrower, and by July 25 had attained a mean width of approximately 40 A.



Variations in widths of emission bands.

Radial Velocities

It has already been pointed out that accurate values of radial velocity cannot be determined from objective prism spectra, but good approximations can be made by the method about to be described. In the case of absorption lines the displacements are so great that errors of the method amount only to a small percentage of the quantity measured, while for emission bands there are, in any circumstances, considerable difficulties in setting (due to their broad nebulous character) which involve larger errors than those due to the method employed. In either case, therefore, it seems that the results obtained may have some value. It may also be pointed out that these results are in agreement with those so far published from other observatories, and that independent measures made on different plates taken on one night indicate only a small probable error inherent in the method.

A number of spectra of stars which have a very small radial velocity were taken under similar conditions to those under which the nova plates were taken. As many lines as possible were measured over the range $H\beta$ to $H\delta$, and the usual Cornu-Hartmann reduction formula computed for each plate. The two constants log C and λ_0, and the curve of errors, were found to be in very good agreement (*i.e.* the scales of all the plates were approximately the same). The average values were assumed to be correct for the nova plates, and the final constant (s_0) was computed for each plate from the measured positions of the interstellar [H] and [K]. Accurate values of the wave-lengths of these lines were kindly supplied from the Solar Physics Observatory, Cambridge, by Dr. A. Beer. A graphical method was also employed in which a plot of wave-length against scale reading (in mm.) was made, using the [K] line as origin. From the curve thus obtained the displaced wave-length of any line corresponding to a known scale position could be found, and from this value the Doppler displacement was computed. This gave results in good agreement with the first method, and the mean of the two was adopted as the final value for each line.

An attempt to estimate the probable errors of the results was made by taking, for any one night, residuals from the mean value of the radial velocity, including results from different plates when available. The metallic lines (other than *Ca*II) and hydrogen lines were treated as separate groups, since their true velocities could not be assumed to be the same. Calculations for five different nights gave fairly consistent values of $\pm$ 25 km./sec. for the probable error of a single observation (one line on one plate). The probable errors of the means given in Table IV should therefore be less than this according to the numbers of lines used.

It is of course assumed that the displacements are due to Doppler effect, and the results are given in the form of radial velocities for convenience of tabulation and comparison, but it must not be forgotten that other causes may be involved in producing the observed displacements.

The results are given in Table IV for the absorption lines of *H*, *Ca*II and the mean of other metallic lines (chiefly *Fe*II, *Ti*II, *Mg*II, *Sr*II and *Cr*II). In the case of the last group, velocities were only deduced from lines

which appeared to arise from a single origin, or from two origins closely adjacent in wave-length, in order to avoid errors due to estimating the true centre of a blend. On later dates than those given in the table, measures were not available owing to the lack of sufficient continuous background to show up the interstellar lines. The extremely high values of some of the *H* components will be noticed, and also the apparent velocity differences between the principal *H* component, *Ca*II and other metallic lines. It is difficult to be sure of the reality of these differences on account of the various sources of error which may affect the results, but they persist in all the negatives and—at least in the case of (*H* – *Ca*II)—may be regarded as probably real.

TABLE IV

Absorption-line Velocities

1936 June	20	22	23	24	25
Hβ 1	...	...	− 930	...	...
Hγ 1	...	− 760	− 980	− 890	...
Hδ 1	...	...	− 990	...	...
Hβ 2	−1260	−1680	−1980	−1960	−2010
Hγ 2	−1150	−1570	−1770	−1770	−1830
Hδ 2	−1120	−1590	−1800	−1900	−2030
Hβ 3	...	−3090	−3240	−3270	−3330
Hγ 3	...	−2510	−3040	−3110	−3270
Hδ 3	...	−3380	−3120	−3270	−3310
Mean *H*1	...	− 760	− 970	− 890	...
„ *H*2	−1170	−1610	−1850	−1880	−1960
„ *H*3	...	−2990	−3130	−3220	−3310
*Ca*II 1	−1080	−1520	−1700	−1870	−1890
[*H*] and [*K*] 2	...	...	−3070	−3210	−3250
Mean metallic lines	−1160	−1640	−1880	−1960	−1970

The emission-line velocities are given in Table V. They are much more difficult to determine, through the width and character of the bands and their complex structure, but the results show a systematic progression of velocity from positive to negative values, both for *H* and metallic lines. The former develop from + 160 km./sec. on June 22 to − 70 km./sec. on June 25, which seems difficult to account for on any current theory of novæ. The possibility of a spurious displacement being caused by the errors inherent in the method must not be overlooked, but if the errors discussed above for absorption lines are applicable to the emission lines, the probable errors for the mean *H* values given in Table V are of the order of ± 14·5 km./sec., which is considerably smaller (with one exception) than the values themselves. This, together with the regularity of the progression, suggests that the observed effect is at least partially real. Possibly the effect is caused by

masking of the red edges of the bands by absorption (or unmasking of the violet edges) and is not a velocity effect at all, but there is no doubt as to the progressive change of the apparent displacements. During the course of this work a paper appeared by Bobrovnikov * in which a similar change is described which fully confirms the present results.

TABLE V

Emission-line Velocities

1936 June	22·0	22·9	23·0	24·0	25·0
Hydrogen lines	+160	+60	−10	−60	−70
Metallic ,,	+ 30	0	−40	−60	−30

A partial check on the foregoing results is afforded by the relative displacements of the absorption and emission components of the hydrogen lines. These may be obtained directly, from the dispersion of the instrument at these wave-lengths, without some of the assumptions involved in the preceding method. The results, given in Table VI, roughly confirm the velocities already obtained. If they are corrected for the measured emission velocities, the resulting absorption-line velocities agree fairly well with those of Table IV.

TABLE VI

Relative Velocities (Abs.–Em.)

1936 June	22·0	22·9	23·0	24·0	25·0
Mean *H*1	− 860	−1280	− 980	− 840	...
,, *H*2	−1850	−1990	−1960	−1820	−1880
,, *H*3	−3080	−3240	−3280	−3210	−3290

Summary

A description is given of the principal changes in the spectrum of Nova Lacertæ (1936) as observed with the 12-inch prismatic camera of the Norman Lockyer Observatory.

The changes run through what is usually regarded as a normal course for such stars, commencing with a widened and displaced α Cygni absorption spectrum, followed by various stages of mixed absorption and emission, and ending with a purely emission spectrum of nebular type.

Tables are given of the more prominent absorption and emission lines, with an indication of their characters, widths and dates of appearance.

* *Contr. Perkins Obs.*, No. 6, 1937.

Approximate values of radial velocities were obtained by using the known wave-lengths of interstellar lines, tables being given to illustrate the results. Some of the absorption lines show very high velocities on later dates, while the emission lines show a progression from positive to negative values.

Thanks are offered to Professor F. J. M. Stratton for valuable criticism and advice during the course of this work, to Dr. A. Beer for supplying positions of interstellar [*H*] and [*K*] from Cambridge plates, and to Professor F. I. Blumbach for taking many of the plates which were used in this investigation.

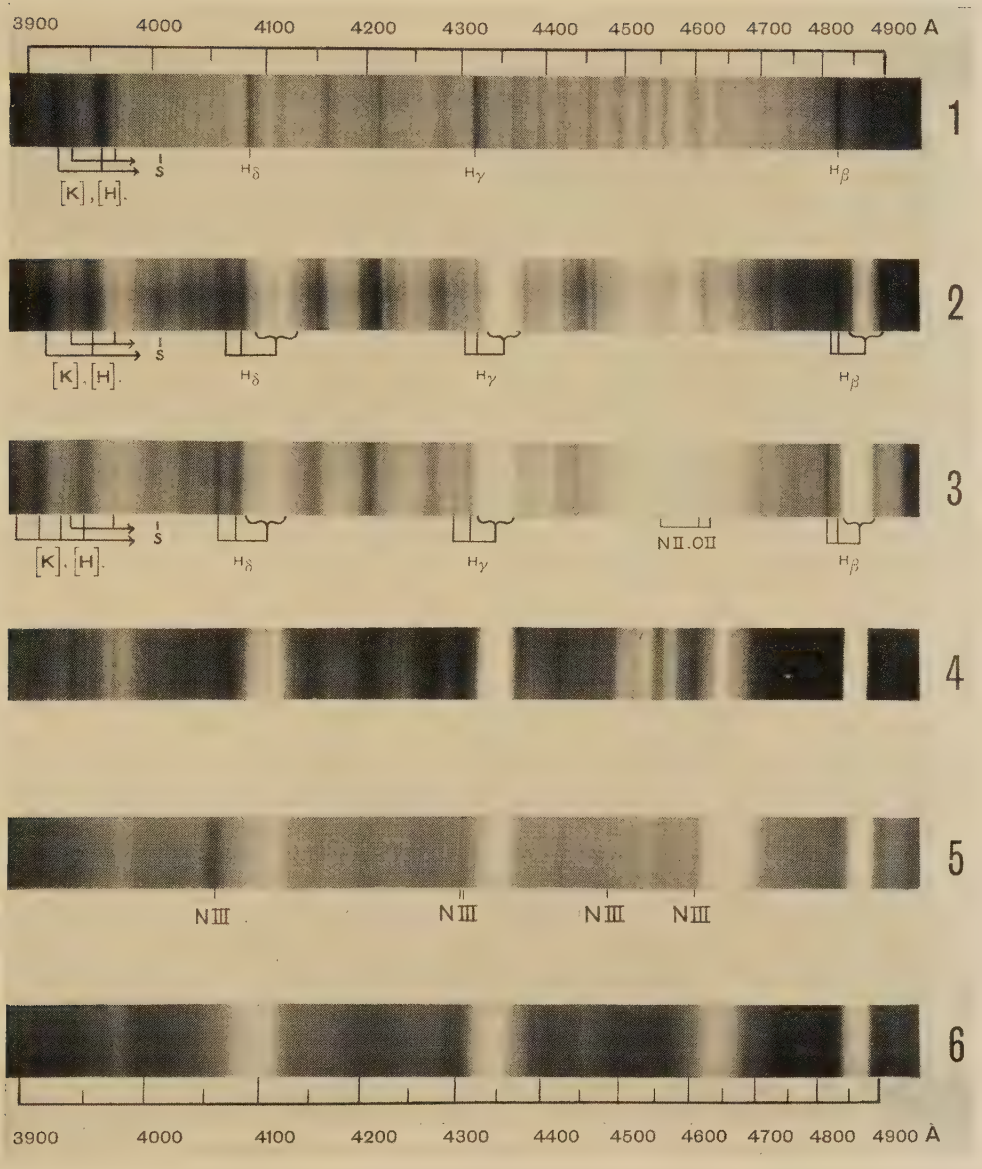
The Norman Lockyer Observatory,
Sidmouth :
1937 *June* 4.

DESCRIPTION OF THE PLATE

(I = Interstellar lines, S = Stellar lines)

1. Nova Lacertæ, 1936 June 20, $02^h\ 22^m - 02^h\ 35^m$ U.T.
   Absorption lines similar to α Cygni but broadened and displaced. Interstellar [*H*] and [*K*] prominent.
2. Nova. June 22, $00^h\ 46^m - 00^h\ 56^m$ U.T.
   Appearance of *H* and metallic emission. Absorption spectrum prominent and showing increased displacement. Appearance of additional Balmer components.
3. Nova. June 24, $00^h\ 44^m - 01^h\ 10^m$ U.T.
   Emission lines strengthening and absorption spectrum further displaced to the violet. Appearance of additional components of [*H*] and [*K*], also a group of *N*II and *O*II absorptions.
4. Nova. June 29, $23^h\ 57^m$ – June 30, $00^h\ 32^m$ U.T.
   Emission spectrum prominent. Absorption lines still present but weakening.
5. Nova. July 7, $22^h\ 55^m$–$23^h\ 55^m$ U.T.
   Metallic absorption spectrum very faint. *N*III absorption lines appear with great intensity. Emission bands 4534 and 4640 have widened and become prominent.
6. Nova. July 20, $00^h\ 31^m$–$01^h\ 42^m$ U.T.
   Typical emission spectrum. Balmer series, 4363 and 4640 prominent. Band at 4534 has faded.

(All the spectra are enlarged and widened by the clepsydra apparatus. A few spurious lines are present.)



Progressive Spectra of Nova Lacertæ, 1936 June–July.

D. L. Edwards and D. R. Barber, Objective Prism Observations of Nova Lacertæ (1936).

A COMPARISON BETWEEN THE RIGHT ASCENSIONS OF THE SECOND GREENWICH CATALOGUE OF STARS FOR 1925·0 AND OF THE FK3.

W. M. H. Greaves, M.A.

(Communicated by the Astronomer Royal)

The present note gives the results of a comparison between the right ascensions of the *Second Greenwich Catalogue of Stars for 1925·0* and of the *Dritter Fundamentalkatalog des Berliner Astronomischen Jahrbuchs*, commonly referred to as the FK3.*

The *Second Greenwich Catalogue of Stars for 1925·0* was published in 1935. It is based on observations made with the Airy transit circle during the years 1922–1930 and consists of two parts. Part I gives places for fundamental stars and for some other stars in Boss's *Preliminary General Catalogue*, together with places for stars used in the Greenwich latitude variation programme, for stars in the Backlund-Hough list north of declination $+64°$ and for 35 stars south of declination $+32°$ used in the international latitude work. Part II contains places for all stars down to magnitude 7·9 on the A.G. scale between the limits 32° and 64° north declination, with the addition of 285 fainter stars in the sparse region of the sky from 12^h to 16^h right ascension. The adopted equinox of the catalogue is the same as that of the FK3, the right ascensions being systematically $0^s·05$ less than right ascensions based on Newcomb's equinox. The positions were reduced to 1925·0 with the aid of adopted proper motions which, for the fundamental and other Boss stars, were Boss's proper motions. The epochs of observation are so close to 1925·0 that the effect of possible slight errors in the adopted proper motions is negligible.

There are 587 stars south of 80° north declination common to the first part of the Greenwich Catalogue and to the FK3, and of these 129 were observed at both upper and lower culminations at Greenwich. The differences in right ascension FK3 *minus* Greenwich 1925_2 were formed for each star, the observations at upper and lower culminations being treated separately.† The following weights were assigned:—

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* Published in the *Berliner Jahrbuch* for 1936. A complete publication with a detailed introduction is promised.

† The Greenwich Catalogue as printed gives only combined results. In view of the small but definite systematic differences that have been found, it is proposed to publish an addendum giving the separate results for above and below pole. The stars affected are all stars north of 64° north declination in addition to the azimuth stars. The addendum will be incorporated in the next Greenwich Catalogue.

Stars observed more than 30 times at Greenwich . . Weight 2
Stars observed less than 5 times at Greenwich * . . ,, $\frac{1}{2}$
Other stars . . . . . ,, 1

The differences FK3 *minus* Greenwich 1925_2 were multiplied by cos δ for each star, grouped according to declination, and meaned over the whole 24 hours of right ascension. The resulting unsmoothed values of $\Delta\alpha_\delta \cos\delta$ are given in Table I. These quantities were then smoothed graphically and multiplied by sec δ, thereby producing the smoothed values of $\Delta\alpha_\delta$ given in Table II (which also incorporates a small additional correction †). For stars observed at lower culmination the quantities $180° - \delta$ are entered in the declination columns of the tables (but for these results cos δ was taken as *positive* in forming the products $\Delta\alpha_\delta \cos\delta$).

TABLE I

Unsmoothed Values of $\Delta\alpha_\delta$ *cos* δ. *FK3* minus *Greenwich* 1925_2

δ	$\Delta\alpha_\delta \cos\delta$	δ	$\Delta\alpha_\delta \cos\delta$	δ	$\Delta\alpha_\delta \cos\delta$
°	s	°	s	°	s
130·4	−·020 *	57·6	−·016	13·1	−·002
120·3	−·016 *	52·5	−·015	7·6	+·005
112·9	−·014 *	47·6	−·009	3·0	+·015
107·4	−·012 *	42·4	−·009	− 2·2	+·007
102·5	−·003 *	37·8	−·014	− 7·5	+·012
77·3	−·008	32·3	−·015	−12·3	+·012
72·6	−·007	27·8	−·009	−20·0	+·014
67·1	−·013	22·8	−·004	−23·5	+·01
62·3	−·013	17·7	−·008		

* *For these lower culmination results cos* δ *was taken* positive *in forming the products* $\Delta\alpha_\delta \cos\delta$.

TABLE II

Smoothed Values of $\Delta\alpha_\delta$. *FK3* minus *Greenwich* 1925_2

δ	$\Delta\alpha_\delta$	δ	$\Delta\alpha_\delta$	δ	$\Delta\alpha_\delta$
°	s	°	s	°	s
135	−·026	70	−·031	20	−·006
130	−·028	65	−·031	15	−·002
125	−·030	60	−·030	10	+·002
120	−·031	55	−·026	5	+·008
115	−·032	50	−·020	0	+·010
110	−·031	45	−·017	− 5	+·011
105	−·025	40	−·016	−10	+·012
100	−·011	35	−·015	−15	+·013
80	−·034	30	−·013	−20	+·014
75	−·031	25	−·010	−25	+·014

The smoothed values of $\Delta\alpha_\delta \cos\delta$ show a decrease from the equator to

* A few results for stars observed at lower culmination for which only one observation per star had been obtained were not used.

† See subsequent footnote.

$\delta = 30°$, an increase from $\delta = 60°$ to $\delta = 80°$, and a decrease for the stars observed at lower culmination from $\delta = 100°$ to $\delta = 135°$.

In the subsequent analysis the smoothed values of $\Delta\alpha_\delta \cos\delta$ were subtracted from the individual values for each star, thereby forming residuals $R_\alpha \cos\delta$. On grouping the R_α's into four groups, viz. right ascensions 0^h to 6^h, 6^h to 12^h, 12^h to 18^h, and 18^h to 24^h, and plotting against tan δ (the stars observed at lower culmination being entered in the groups 12 hours different in R.A., and tan δ taken as negative), it was seen that the residuals R_α varied linearly with tan δ within each group, the slope being negative (*i.e.* the R_α's diminishing with increasing values of tan δ) for the first two quadrants and positive for the last two. This indicates, relative to the FK3, a periodic error in the Greenwich determinations of azimuth. With the object of evaluating this more precisely, the stars were divided into 24 hourly groups. For each of these groups each star gave an equation of condition of the form

$$\mu + \nu \tan\delta = R_\alpha \qquad (1)$$

As before, the stars observed at lower culmination were entered in the groups 12^h away in R.A. (*e.g.* such stars with right ascensions between 0^h and 1^h were entered in the 12^h to 13^h group) and for them tan δ is to be taken as negative.

On multiplying (1) by cos δ there results

$$\mu \cos\delta + \nu \sin\delta = R_\alpha \cos\delta \qquad (1')$$

For each hourly group the equations of condition in the form (1') * were solved by least squares. The resulting values of μ and ν are given in Table III, in which the unit is one ten-thousandth of a second of time.

TABLE III

Values of μ and ν

Unit = $0^s.0001$

R.A.	μ	ν	R.A.	μ	ν
h h			h h		
0–1	+50	− 34	12–13	−63	+ 61
1–2	+ 9	+ 12	13–14	−22	+ 61
2–3	+22	− 28	14–15	−91	+121
3–4	+82	− 25	15–16	− 7	+ 8
4–5	+91	− 75	16–17	−12	− 6
5–6	−35	−156	17–18	−18	+ 67
6–7	−35	− 46	18–19	− 2	+ 20
7–8	+ 3	−132	19–20	−13	+ 34
8–9	−47	− 71	20–21	+80	+ 52
9–10	+10	− 44	21–22	+28	+127
10–11	−15	− 87	22–23	+85	+ 80
11–12	−85	− 4	23–24	− 8	+ 76

* In which for the stars observed at lower culmination cos δ was taken as positive and sin δ as negative.

The periodic nature of the quantities μ and ν is obvious by inspection. Harmonic analysis gave the following expressions *:—

$$\left.\begin{aligned}\mu &= +0^s\cdot0049\cos\alpha + 0^s\cdot0005\sin\alpha - 0^s\cdot0005\cos 2\alpha - 0^s\cdot0003\sin 2\alpha\\ \nu &= +0^s\cdot0015\cos\alpha - 0^s\cdot0078\sin\alpha + 0^s\cdot0036\cos 2\alpha + 0^s\cdot0004\sin 2\alpha\end{aligned}\right\} \quad (2)$$

On subtracting the values of μ and ν given by the harmonic formulæ (2) from the individual hourly values given in Table III, the resulting residuals gave a probable error for an hourly value of $\pm\ 0^s\cdot0027$ for μ and $\pm\ 0^s\cdot0029$ for ν. Taking $\pm\ 0^s\cdot0028$ as the p.e. of an hourly value, the resulting probable error of the Fourier co-efficients in the formulæ (2) is $\pm\ 0^s\cdot0008$.

We now write

$$\Delta\alpha_\alpha = \mu + \nu\tan\delta \quad (3)$$

where μ and ν have the values given by (2).

The values of $\Delta\alpha_\alpha$ are given in Table IV, it being understood that for stars observed at lower culmination $\alpha + 12^h$ is to be substituted for α and $\tan\delta$ taken negative.

TABLE IV

Values of $\Delta\alpha_\alpha$. *FK3* minus *Greenwich* 1925_2

R.A.	$\Delta\alpha_\alpha$			R.A.	$\Delta\alpha_\alpha$		
h	s	s		h	s	s	
0	+·004	+·005	tan δ	12	−·005	+·002	tan δ
1	+·004	+·003	,,	13	−·005	+·004	,,
2	+·004	·000	,,	14	−·005	+·005	,,
3	+·004	−·004	,,	15	−·004	+·005	,,
4	+·003	−·007	,,	16	−·003	+·005	,,
5	+·002	−·010	,,	17	−·001	+·004	,,
6	+·001	−·011	,,	18	·000	+·004	,,
7	·000	−·011	,,	19	+·001	+·005	,,
8	−·002	−·010	,,	20	+·003	+·005	,,
9	−·003	−·007	,,	21	+·003	+·006	,,
10	−·004	−·004	,,	22	+·004	+·007	,,
11	−·005	−·001	,,	23	+·004	+·006	,,

For stars observed at lower culmination use $12^h + \alpha$ *as argument instead of* α, *and take* $\tan\delta$ *negative.*

The complete systematic difference $\Delta\alpha$ between the right ascensions of the *Second Greenwich Catalogue of Stars for 1925·0* and the FK3 (in the sense FK3 *minus* Greenwich 1925_2) is given by

$$\Delta\alpha = \Delta\alpha_\alpha + \Delta\alpha_\delta \quad (4)$$

where $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$ are to be extracted from Tables IV and II. They represent corrections which are applicable to the right ascensions given in both parts of the Greenwich Catalogue in order to reduce them to the system of the FK3. As far as can be judged, there are no further systematic differences in right ascension.

* The actual analysis gave constant terms amounting to $+0^s\cdot0005$ for μ and $-0^s\cdot0010$ for ν. These values have been subtracted from the original hourly values of μ and ν, derived from the 24 least-square solutions, and Table III as now printed does not include them. The quantity $+0^s\cdot0005 - 0^s\cdot0010\tan\delta$ was added to the values of $\Delta\alpha_\delta$ as obtained from the original smoothing and is now included in Table II.

The presence in the Δa_α's of periodic terms involving the tangent of the declination may be attributed to a periodic error in the determination of the azimuth of the Airy transit circle. The total range in the values of ν given by (2) is from $+0^s.007$ at 22^h to $-0^s.011$ at 6^h and 7^h, and this is to be interpreted as a periodic error, relative to the FK3, of the Greenwich azimuth determinations, the annual range in arc being 0.018×15 sec. (latitude) $= 0''.43$.

The origin of a variation such as this is obscure.* From a preliminary comparison of the *First Greenwich Catalogue of Stars for 1925·0* with the FK3, Mr. Cullen has found a periodic azimuthal term in the R.A.'s of the same order of magnitude as, but opposite in phase to, the term involving tan δ found in the present note. For the comparison FK3 *minus* Greenwich 1925_1, the residuals R_α (see above), when plotted in 6-hour groups against tan δ, exhibit a positive slope for the 0^h–6^h and 6^h–12^h quadrants and a negative slope for the other two quadrants. There is thus a definite difference in this respect between the two Greenwich Catalogues 1925_1 and 1925_2. It seems to be associated with a change in the system of the azimuth stars. For the first of the two catalogues (1925_1) the azimuths were based on the observations of nine stars situated within 4° of the north pole. In 1922, at the commencement of the second catalogue (1925_2), the azimuth list was extended down to declination $+45°$ and now comprises 79 stars. The right ascensions adopted for these stars in 1922 were extracted from the previous catalogue and were based mainly on observations of Polaris at upper and lower culmination. They were subsequently revised by right ascensions depending on observations of the brighter stars on the new list at upper and lower culmination. By the year 1925 it was found that the revision had altered the right ascensions appreciably,† the revised R.A.'s being (in addition to a constant difference) in excess of the originally adopted values between right ascensions 0^h and 12^h, and less than the adopted values between 12^h and 24^h. These revised places were used over the period 1925–1930. In brief, the change of azimuth system whereby the determinations of azimuth were based on observations of stars down to declination $+45°$, instead of being based mainly on Polaris observations, seems to have introduced a marked change in the periodic azimuthal term. But the precise instrumental origin of the change is obscure.‡

* It may be noticed that considerably greater azimuthal terms have been found in a few of the older catalogues. See the detailed systematic corrections given in Appendix III of Boss's *Preliminary General Catalogue*.

† See p. viii of the introduction to the *Second Greenwich Catalogue of Stars for 1925·0*.

‡ A *possible* contributory cause would be a diurnal oscillation in azimuth with a seasonal change in its amplitude. It may be noticed that if such a variation existed its effect on the derived right ascensions would be largely eliminated by the use of the new system of azimuth stars. For fundamental azimuths are now often determined by the combination of a day observation of a star near the zenith with a night observation of the same star at lower culmination, and an azimuth error so determined is correctly applicable to the night observations. But when azimuths are based on transits of stars near the pole there would be no such elimination.

Summary.—The systematic differences in right ascension between the Second Greenwich Catalogue for 1925·0 and the FK3 have been determined. The $\Delta\alpha_\alpha$ values include periodic terms corresponding to an annual error, with a total range of 0″·4, in the determination of azimuth. A similar effect, but of opposite phase, is present in the previous Greenwich Catalogue, and the change between the two catalogues appears to be associated with a change in the system of determining azimuth, whereby the list of azimuth stars was extended from the neighbourhood of the pole to south of the zenith.

I am indebted to Mr. R. T. Cullen for stimulating help in the course of many discussions.

Royal Observatory,
Greenwich :
1937 *October* 26.

ON STATISTICALLY STEADY DISTRIBUTIONS IN ASTRONOMY.

Harold Jeffreys.

It is well known that a system of bodies released in such a way that the details of encounters between them cannot be predicted will tend, in certain conditions (such as being confined to a limited region), to a state specified by the equipartition of energy and Maxwell's distribution of velocities. The usual arguments given are those of Boltzmann and Gibbs ; I have recently pointed out some defects in these arguments, but a detailed discussion in terms of the theory of probability has again led to a close approximation to the usual rules.* In some astronomical problems connected with star-streaming and the structure of clusters, however, it has appeared that close encounters are so rare that there has not been time for any approach to a Maxwellian distribution to be established. Quantitative estimates are given in Jeans's *Astronomy and Cosmogony*. The general field of force, however, has had time to affect the motion of each body considerably ; thus Jeans estimates that each star in the galaxy revolves about the centre in about 150 million years. Jeans has given a theorem concerning steady distributions possible when the general field is taken into account but close encounters are not.† He and several other authors, including myself, have made applications of this theorem ; but I have recently come to doubt whether the theorem is valid, and it certainly will not permit the use that I have made of it.

Jeans denotes the number of stars in the typical element by N, so that

$$N = f(u, v, w, x, y, z)\,dudvdwdxdydz. \tag{1}$$

At a later date these stars occupy an element $du'dv'dw'dx'dy'dz'$. But their number is unaltered, and we shall have

$$f(u, v, w, x, y, z)\,dudvdwdxdydz = f(u', v', w', x', y', z')\,du'dv'dw'dx'dy'dz'. \tag{2}$$

But it follows easily from Hamilton's equations that

$$\frac{d}{dt}\frac{\partial(u', v', w', x', y', z')}{\partial(u, v, w, x, y, z)} = 0 \tag{3}$$

whence the differentials are equal and

$$f(u'v'w'x'y'z') = f(u, v, w, x, y, z). \tag{4}$$

If the system is steady, f does not involve the time explicitly ; it appears therefore that the necessary and sufficient condition for steady motion is

* *Proc. Roy. Soc.*, A, **160**, 325–348, 1937.

† *M.N.*, **76**, 78–79, 1915.

that f shall be expressible entirely in terms of functions of the co-ordinates and velocities that do not change along a dynamically possible path of a particle; in other words, of the first integrals of the equations of motion.

Now several objections may be made both to the argument and to the result. The conclusion would mean that if N is the number of stars in the element $dudvdwdxdydz$ at time t, it will also be the number in that element at time $t+dt$; the number entering the element must be exactly equal to that leaving it. This could hold only for elements of very special sizes and shapes with special initial conditions, so that whenever a star leaves another enters at the same moment. In general the numbers will differ on account of sampling errors. It could, however, be interpreted to mean either (1) that in a given state of knowledge the expectations of the numbers entering and leaving are equal, or (2) that these numbers are so large that sampling errors are negligible in comparison with them. Now (1) equality of the expectations does not prove that the numbers are even approximately equal. Consider the following problem. Two boxes, A and B, contain the same number of balls, n. We toss a coin. If it comes down heads we will transfer all the balls from A to B; if tails, all from B to A. The expectation of the number of balls in A after the transfer is n; but the actual number will be either 0 or $2n$. The expectation n arises from the fact that these are two untested possibilities and are equally probable on the data at present available. The fact that the expectation of the change of the number of balls in A is zero is quite consistent with the fact that the actual change will be either $+n$ or $-n$. To complete the argument it would be necessary to show that the probability of different changes was closely concentrated about zero change. Again, (2) if the numbers entering and leaving are large, the velocity-position element must be large or the element of time long; but in that case we are not entitled to treat du, dx and dt as differentials that can tend to zero. Attention to the magnitudes involved would be needed before we could draw any conclusions about the sizes of the elements such that the result could be expected to hold as an approximation. Neither interpretation, therefore, is satisfactory as the matter stands.

It should be noticed that the elements $dudvdwdxdydz$ and $du'dv'dw'dx'dy'dz'$, though equal in volume, are different in shape, if we imagine the velocity and position components plotted in six dimensions. If the element at time t is rectangular, that at time $t+dt$ will be extended in some directions and contracted in others, the maximum proportional changes being inclined to the axes, and after a moderate time it will become a long narrow filament. If we restrict ourselves to rectangular elements, any element at time t' that corresponded to a part of the original element at time t will have to be smaller and smaller, until the number of stars in it becomes 0 or 1, and any further statistical argument becomes meaningless. The difficulty is therefore the same as the one that I pointed out in the Gibbs method. An element of the original size and shape at time t' will average the values of f over a wide range at time t. It is really to the latter fact that the possi-

bility of any inferences about the ultimate state of a gas is due, and it is just this fact that the Gibbs treatment of gases and Jeans's treatment of stellar systems omit to consider.

The difficulty about sampling errors makes it desirable to consider long intervals of time, if possible, and to think in terms of the probability that there will be a star in a given small element at any time. In order that a star may be within the element it must have given position and velocity components, and in a fixed field of force certain functions of these must always be the same. Hence f may be a function of these functions. Other functions may not be always the same, and f, after a long time, will be an average of the initial values over a long range of these functions, the variations of which will therefore average out. The first integrals of the equations of motion, considered by Jeans, are functions of the first type, but they are not necessarily the only ones. Thus in the problem of a single dominating central mass with small bodies moving about it, the energy and angular momentum components of a particle remain constant throughout the motion, but these specify only four orbital elements, the plane of the orbit (requiring two parameters), the major axis and the eccentricity. The longitude of perihelion and the time of perihelion passage, however, also remain constant. On Jeans's analysis these would be irrelevant, but it can be seen that they are not. Suppose that the system consists only of a swarm of particles projected along the same circular orbit over an interval of time short compared with the period of revolution; they will maintain their distances for ever, and we shall have a steady motion in the usual sense understood in dynamics, that the ignorable velocities and the non-ignorable co-ordinates all remain constant. (It would not be steady if mutual attractions were taken into account, but by hypothesis these are neglected.) Thus the time of perihelion passage is not irrelevant to the distribution function in a steady motion.* Again, suppose that a set of particles are projected at equal intervals of time along the same elliptic orbit. They will follow one another round the orbit for ever, and we shall have a steady motion such that the maximum extension of the system is different in different directions and the radial motions on one side are all inwards and on the other all outwards. It will be possible to choose finite regions such that the number of particles in them remains exactly constant—an extreme of steadiness not contemplated in the statistical argument. Such a case suggests the special cases in kinetic theory where special initial conditions prevent interchange of energy, and the Maxwellian state would never be reached, but there is a fundamental difference. In the latter cases a small and unpredictable change of the initial conditions would introduce collisions of the excluded types, and the Maxwellian state would still be reached. In the present one the neglect of mutual influence between the particles is fundamental to the problem, and this escape is impossible. Small variations

* But if there are slight differences in the initial velocities these will lead, in a long enough time, to considerable differences of longitude, and the chance of a particle in a given range of longitude will become uniformly distributed. This is typical of the kind of averaging needed.

of the initial conditions would still leave the motion permanently asymmetrical. More complicated systems could be built up by projecting along different orbits but with the perihelia all along the same line; the same asymmetry would persist.

By applying Jeans's theorem to this problem I found that steady states would necessarily be symmetrical, but these examples show that this result does not follow from the hypotheses, and it appears that the theorem cannot be true, at least if the first integrals admitted are restricted to those specified by Jeans. It is indeed doubtful whether the last restriction is justifiable. When the initial conditions for one particle are specified, all the position and velocity components are specified for all time as functions of the time. Thus elimination of the time between any pair of these will lead to a relation independent of the time, which would apparently be a first integral of the equations of motion in the sense contemplated in the proof of the theorem, though not in the sense contemplated in particle dynamics.

In a paper written at the beginning of 1915 and referred to by Eddington * I constructed steady motions applicable to this problem; the method was to assume distributions of the orbital elements and work out by integration the distribution function for the position and velocity at any instant. Symmetry was introduced as a special assumption and achieved by arbitrarily taking the times of passage through a given longitude and the longitudes of perihelion as uniformly distributed. This assumption, however, seemed to me unsatisfactory, and when Jeans's work was available it appeared to provide a much better procedure; consequently my published treatment of the problem † rests on his theorem, and my first one has long since been destroyed. It appears, however, that without a special assumption there is no reason to expect symmetry.

If the particles are not all at the same mean distance their mean motions will differ; if the times of passing a given longitude are not uniformly distributed, the longitudes of any concentrations of density will increase at different rates for different distances, and the motion will not be steady. It is therefore necessary for steady motion, if the mean distances differ, that the times at a given longitude shall be uniformly distributed. There does not, however, appear to be any such restriction on the longitude of perihelion, and a permanent asymmetry seems still to be possible.

In the case of a star cluster the orbits will not in general be closed curves; nodes and pericentres will revolve at different rates, depending on at least the mean distance. This apparently makes any permanent asymmetry impossible in a steady motion, and I think that Jeans's final conclusion, that a steady motion in this case must be symmetrical about an axis, is correct.

In a gas, composed of molecules that behave like elastic solid particles or that repel according to a high negative power of the distance, close encounters play the chief part in changing the directions of motion and promoting the applicability of the gas laws. In a stellar system, where the chief mutual influence is gravitation, this does not appear to be true.

* *M.N.*, **75**, 368, 1915.
† *M.N.*, **77**, 85–94, 1916.

If regions of excess and defect of mean density of order σ are separated by distances of order l, the accelerations produced by one such departure on the near and far sides of a neighbour will differ by quantities of order $f\sigma l$, where f is the constant of gravitation. The differences of acceleration produced will therefore be greater the more widespread the irregularities are. The direct effect would be to promote condensation, but with transverse relative velocities they will lead to changes in the directions of motion of the kind needed to produce equipartition. Thus the gas laws may be established most rapidly for the large-scale irregularities, not the small ones, as for a true gas. It therefore seems possible that it would be legitimate to treat star clusters as gaseous masses in discussing steady motions, so long as we do not apply this treatment to regions too small compared with the whole size of the system. In that case the whole of Jeans's results would follow. Attention to the appropriate ratio of specific heats would be needed.

I have applied Jeans's theorem to the steady motion of a system of small particles about the Sun and a planet in a circular orbit.* In this case the only first integral of the equations, in the usual sense, is the Jacobi integral, and it led to the conclusion that the mean motion at any distance is equal to that of the planet. Thus beyond a certain distance the mean velocity exceeds the parabolic one, and the particles would escape. The surprising result that such a system can rotate steadily like a rigid body is, however, avoided if the theorem is untrue, which I believe to be the correct solution. It has been suggested that the theorem does not apply to rotating axes, but the proof involves the axes only through Liouville's theorem, which I proved for axes in uniform rotation.†

For Saturn's rings I found a solution with each particle moving in a nearly circular orbit about the primary with the appropriate velocity. There is here a further difficulty, since I showed that a particle must undergo several collisions per revolution. The neglect of close encounters is therefore unjustified, and there has been time to attain a Maxwellian state. This, however, would not be a permanent steady motion, since it is not a rigid body rotation, and statistical theory would lead to analogues of viscosity and heat conduction, and therefore to secular changes. The appropriate treatment would be to adopt the equations of motion for a gas, including the dissipative terms, and to investigate the time-scales of the various changes that will occur. If the viscosity terms are small, the solution for a perfect gas may be taken as a first approximation, but it will vary slowly with the time. The discussions of Laplace, Kowalewsky and others on a gaseous ring all appear to assume that the ring would rotate like a rigid body. This is equivalent to saying that the viscosity terms are much more effective in annulling differences of rotation than the differences of the attraction of the primary at different distances are in producing them. It would therefore require for its justification an investigation of the orders

* *The Earth*, 1929, 52–56.

† In the determinant in my equation (5) the second element in the first row should be $2ndt$, the first in the second row $-2ndt$. This correction does not affect the result that the determinant $=1+O(dt^2)$.

of magnitude of the respective terms, which, as far as I know, has not been provided.

It appears certain that collisions are so frequent that the gas laws will apply. It is also established that the ring rotates in different periods at different distances; hence the hypothesis that a ring satisfying the gas laws would revolve like a rigid body is contradicted by the present state of the ring. Viscosity must, however, be producing secular changes. The velocity of agitation will also correspond to a virtual temperature. This will not be the ordinary temperature as measured by a bolometer, since this would correspond to the molecular agitation. We are concerned with the agitation of the complete particles, and it is only in the final state, where equipartition between molar and molecular energy has been reached, that these could be equated. The effective temperature need not be uniform, so that the equation of heat conduction must also be expressed. There is one complication that would be absent from a perfect gas, that the collisions would be imperfectly elastic. There is experimental evidence that at sufficiently small velocities of impact collisions become perfectly elastic as regards the normal velocities,* but this cannot hold for tangential velocities. If friction decreased more rapidly than the normal force, light bodies would be unable to rest on an inclined plane; we cannot therefore assume that the conversion of molar into molecular energy by collisions of rough bodies will cease, however gentle the collisions may become, since the coefficient of friction must remain finite. The molecular energy will be radiated away as dark heat. Thus there will be a continual loss of energy proportional to the frequency of collision and the square of the velocity of agitation. This does not appear in the gas problem, and it is this that explains how the rings became thin. It might appear possible for the rotations of the particles to adjust themselves so that there would be no velocity of slip before encounters, and that loss by friction could be avoided; but collisions will not be all in the plane of revolution, and no rate of rotation could satisfy the condition of no slip before collision for collisions in all planes.

The solution will obviously be difficult, but the first need is that the problem should be correctly stated.

* J. P. Andrews, *Proc. Phys. Soc.*, 43, 1–25, 1931.

VARIABLES IN KAPTEYN'S SELECTED AREAS 2–19.

E. A. Baker, D.Sc.

(A Communication from the Royal Observatory, Edinburgh)

II. *Variables in Areas* 3 *and* 4

This paper follows the first of the series * in giving information about the variables in the selected areas which rise above the 14th magnitude (photographic), so far as these could be discovered by searching, under the blink-microscope, eight pairs of plates of each area. As in the last paper, the maps are ruled with rectangular co-ordinates, the centre of the area ($4^h\ 8^m$, $+75°\ 0'$, 1900·0, for area 3, and $8^h\ 4^m$, $+74°\ 50'$, 1900·0, for area 4) being given the co-ordinates 500′, 500′, and the abscissæ being parallel to the meridian through it at epoch 1900·0. The times of observations are geocentric and are liable to errors of several minutes owing to the rapid changes in the transparency of the Edinburgh sky.

TABLE I

Variables in Selected Area 3

Star	Unnamed	X Cam	Unnamed
Harvard Designation	034074	043274	043673
R.A. 1900·0	$3^h\ 40^m\ 30^s$	$4^h\ 32^m\ 47^s$	$4^h\ 36^m\ 16^s$
Dec. 1900·0	$74°\ 15'\ 32''$	$74°\ 54'\ 34''$	$73°\ 23'\ 48''$
Period	4^d·404	143^d	560^d
Ptg. Max.	11·6	10	11·5
Ptg. Min.	12·5	15	12·9
Ptg. Mag. at J.D. 242			
7152·408	11·9	10·6	12·6
7154·449	11·9	10·8	12·7
7159·405	11·9	11·4	12·6
7160·394	12·3	11·3	12·6
7362·413	12·1	12·6	12·0
7486·466	12·1	14·9	11·7
7504·405	11·9	13·7	11·5
7665·542	12·0	10·1	12·5
7744·419	11·9	13·5	12·8
7750·419	12·1	14·2	12·9
7842·527	12·5	10·6	12·0
7887·413	11·6	13·4	12·3
7889·405	12·0	14·0	12·2

* *M.N.*, **97**, 541, 1937.

TABLE I—*continued*

Star	Unnamed	X Cam	Unnamed
Harvard Designation	034074	043274	043673
8107·361	12·4	11·2	11·8
8122·313	12·0	10·3	11·8
8434·457	11·6	10·0	11·9
8487·361	11·7	13·1	11·8
8515·343	11·8	11·2	12·0
8622·387	12·2	14·4	11·6
8653·436	12·4	13·9	11·6
8805·400	11·8	12·5	12·8

Notes on Table I

034074.—This star is a Cepheid, with a typical asymmetric light-curve.

043274 (X Cam).—A well-known and extensively studied variable with alternating high and low maxima. This alternation, as the observations here given show clearly, applies to the whole light-curve, the actual period being 286 days.

043673.—In its combination of an exceptionally long period and a comparatively small range this variable falls into a group of which only one or two stars are at present known, and which can be studied only by a lengthy series of observations. The variable AB Cygni is probably a similar star. The present observations, it may be noted, do not altogether preclude the periods $1 \pm 1/560$ day, the period 0·9982 day in particular giving fair agreement, but such periods are improbable. Visual examination at minimum shows that the star is a little redder than the comparison stars (colour index about 0·3 greater), so that the spectral type is probably K. The light-curve is not well determined, the observations not being well enough distributed over it, but the variations cannot be accounted for by supposing the star to consist of a normal long-period variable with a companion of constant magnitude.

In addition to these three variables and to a number of stars showing probable variations of a few tenths of a magnitude, the star *Bergedorfer Spektral-Durchmusterung, Eichfeld* 3, Nr. 851 ($3^h\ 28^m$, $+75°\ 14'·5$, 1900·0, type Go) is absent from the plate of J.D. 2427632, which shows near-by stars to magnitude 15·5. On the remaining plates the star is normal, and a search made at Harvard Observatory on plates taken within a few hours of this observation leaves little doubt that the disappearance is due to some defect of the negative, which has left no other signs of its presence.

Notes on Table II

073672.—The range of variation and the manner in which the observations are scattered over that range suggest that this star is a Cepheid or cluster-type variable. A search failed to disclose any period greater than two days.

TABLE II

Variables in Selected Area 4

Star	Unnamed	Unnamed	Unnamed	SW Cam	Unnamed	Unnamed	Unnamed	Z Cam.	Unnamed
Harvard Designation	073672	074374	074773	075273	075476	080773	081373	081473	082474
R.A. 1900·0	$7^h\ 36^m\ 41^s$	$7^h\ 43^m\ 28^s$	$7^h\ 47^m\ 29^s$	$7^h\ 52^m\ 19^s$	$7^h\ 54^m\ 31^s$	$8^h\ 7^m\ 36^s$	$8^h\ 13^m\ 35^s$	$8^h\ 14^m\ 1^s$	$8^h\ 24^m\ 18^s$
Dec. 1900·0	72° 43′ 43″	74° 30′ 35″	73° 3′ 10″	73° 41′ 3″	76° 39′ 38″	73° 24′ 11″	73° 4′ 29″	73° 25′ 47″	74° 35′ 22″
Period	(Short ?)	(Short ?)	$6^d.16$	254^d	$2^d.881$?	Irreg.	$5^d.03$?	Irreg.	(Short ?)
Ptg. Max.	13·3	12·7	10·8	11	13·8	13	13·5	10	13·8
Ptg. Min.	14·2	13·7	11·4	16	14·6	15·5?	14·6	14	14·5
Ptg. Mag. at J.D. 242									
7450·374	< 13·5	13·0	11·1	< 14	< 14	14·4?	< 13·8	13·5	< 14
7482·325	14·2	13·7	10·9	< 15·5	14·0	14·2	14·1	10·7	14·0
7510·505	13·6	13·2	11·0	13·8	14·5	15·0	13·9	13·5	14·2
7742·503	14·3	12·7	11·0	< 15	14·7	14·4	13·9	11·2	14·1
7770·361	14·1	13·4	11·2	13·7	14·2	14·0	14·2	13·6	14·1
7895·513	14·1	13·2	10·9	14·2	14·6	14·2	13·8	11·6	14·0
8097·518	14·0	13·2	11·3	11·5	14·6	14·9	13·9	13·5	14·0
8101·554	13·9	13·1	11·3	11·5	14·1	14·9	13·5	13·8	14·3
8122·357	...	13·2	11·1	12·4	13·7	13·8	13·9	13·2	14·0
8130·445	13·7	13·0	11·0	13·1	14·2	14·8	13·9	13·6	13·9
8157·336	13·5	13·7	11·3	15·3	14·0	13·1	13·8	10·8	14·1
8186·324	13·6	13·6	11·1	15·6	14·0	13·6	14·6	13·4	13·9
8191·341	13·9	13·4 ?	11·1	15·3	14·0	15·0	14·2	13·5	14·1
8211·306	13·3	13·4	11·2	< 15·6	13·8	14·9	14·2	13·2	14·1
8465·486	13·8	13·0	11·3	< 15·4	14·0	13·3	13·8	13·9	14·5
8487·406	13·8 ?	13·0	11·1	< 15·4	14·1	13·1	13·9	11·1	13·9
8494·431	13·9 ?	13·0	11·1	< 15	14·0	13·3	13·8	13·8	14·3
8511·528	13·5	12·8	10·8	13·5	14·0	< 14·2	13·8	13·8	14·0
8541·296	13·6	12·8	11·4	12·1	14·3	< 14·3	13·8	10·3	13·9
8570·392	13·8	12·9	11·4	11·2	14·5	14·6 ?	13·8	11·3	13·8
8608·507	13·8	12·8	10·9	11·3	14·2	14·7	14·2	12·2	13·9
8622·437	14·0	13·3	10·9	11·5	14·5	14·1	13·9	12·1	14·5
8632·456	13·6	13·0	11·3	12·2	14·4	< 15·3	13·8	11·8	14·1
8657·451	13·8	13·0	11·3	14·2	14·2	14·4	13·8	10·9	14·1
8792·647	13·3	12·9	11·3	14·6	14·5	14·3	13·6	13·8	14·1

074374.—No period has been found.

074773.—A Cepheid of a normal type, with spectral type A3.

075273 (SW Cam).—The star is noteworthy for its early spectral type Go–G5. The light-curve is also unusual, the fall being as rapid as the rise.

075476.—The star is probably a short-period Cepheid or cluster variable, but the period suggested is not altogether satisfactory.

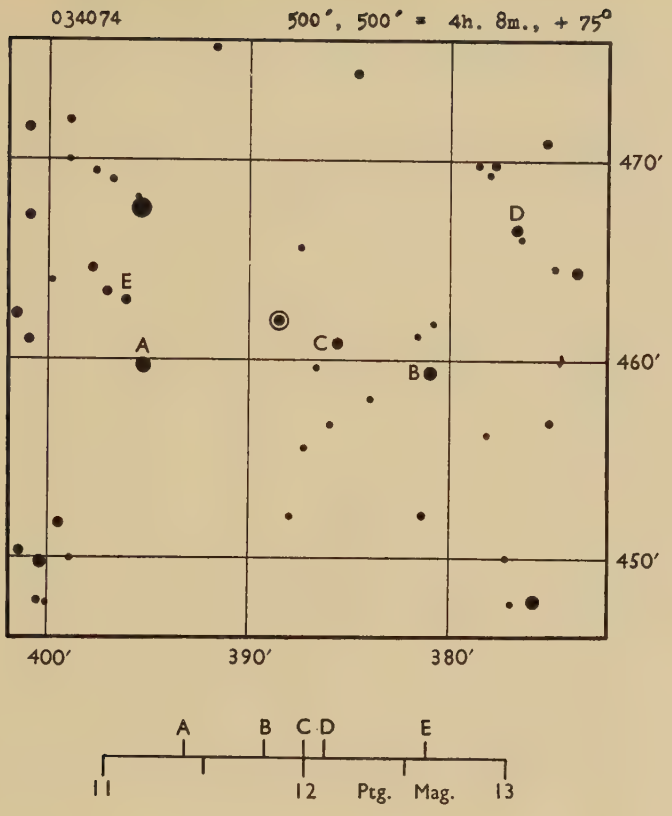
080773.—An irregular or semi-regular variable.

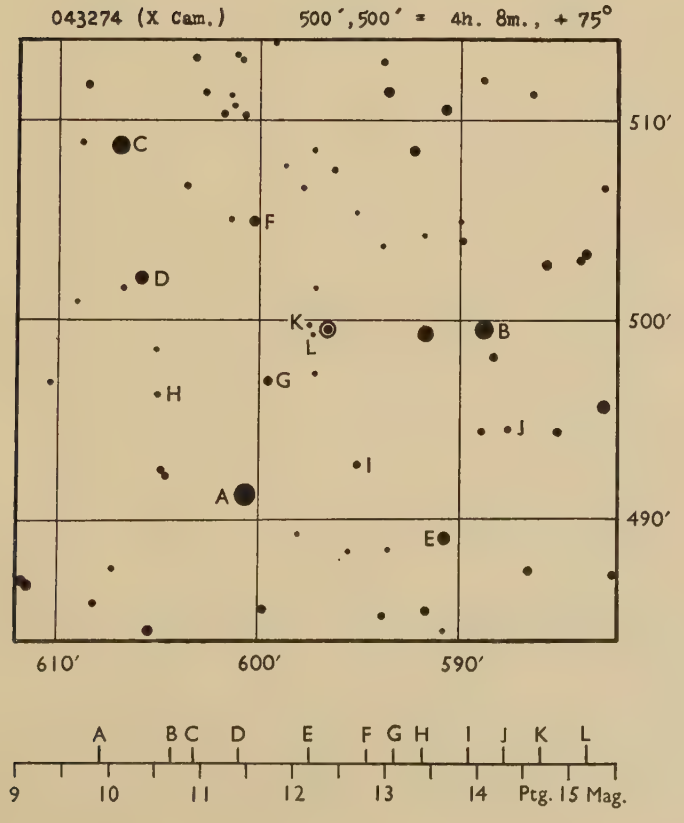
081373.—This is obviously an eclipsing binary; but the period may turn out to be other than that given.

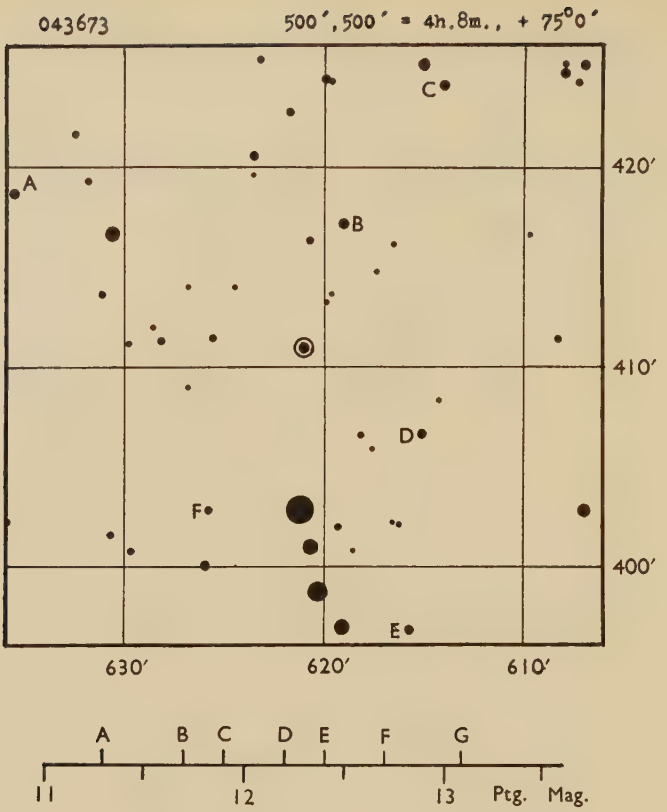
081473 (Z Cam).—A semi-regular variable with a period of the order of 23 days, and spectral type F or G.

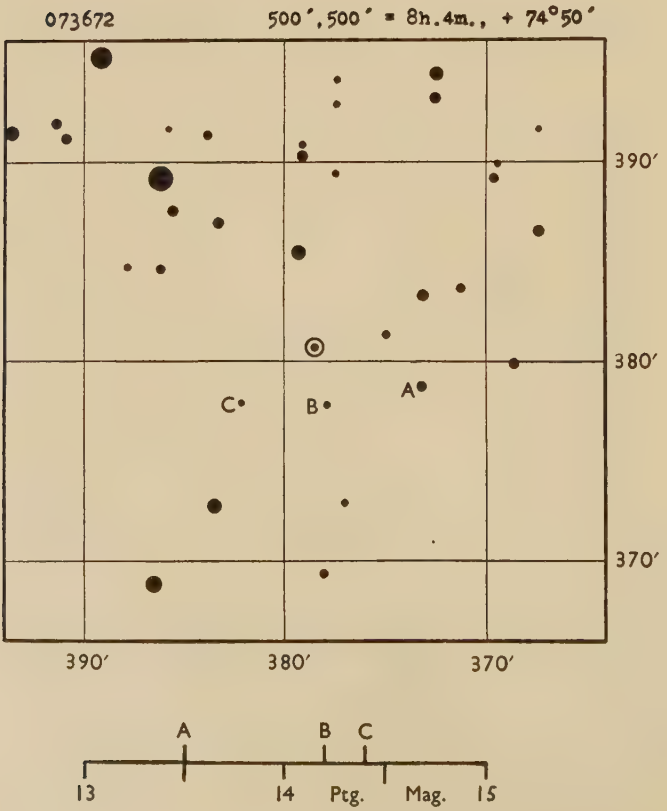
082474.—Probably an eclipsing star of small range. The observations are as yet insufficient to give the period.

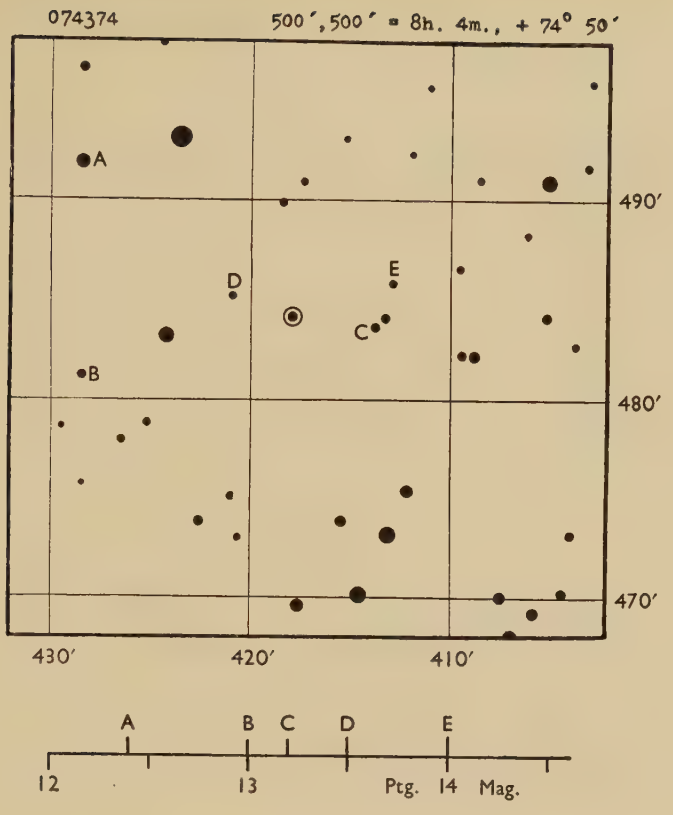
Y Camelopardalis.—It was thought superfluous to give a map of the region of this well-known eclipsing binary, since photographic magnitudes of comparison stars have already been published by Harwood (*H.A.*, 84, 38) and a photographic chart of the region by Ceraski (*Moscow Annals*, II, Vol. 5, Appendix). The variable is, moreover, only just within the area covered by the plates, the most useful comparison star, Harwood's *e*, mag. 10·48, not being covered. The star reached magnitude 10·3 or 10·4 on all plates except those of J.D. 2428186 (mag. 10·5); 2428622 (mag. 11·9); and 2428632 (mag. 11·8). The latter two observations fix the epoch of minimum as J.D. 2428632·40 within 0·05 day, in agreement with the predicted minimum J.D. 2428632·37 (*Rocznik Astronomiczny*, Nr. 15, Cracow, 1937). The position of the star was found to be R.A. $7^h\ 27^m\ 46^s$, Dec. $+76°\ 17'\ 51''$.

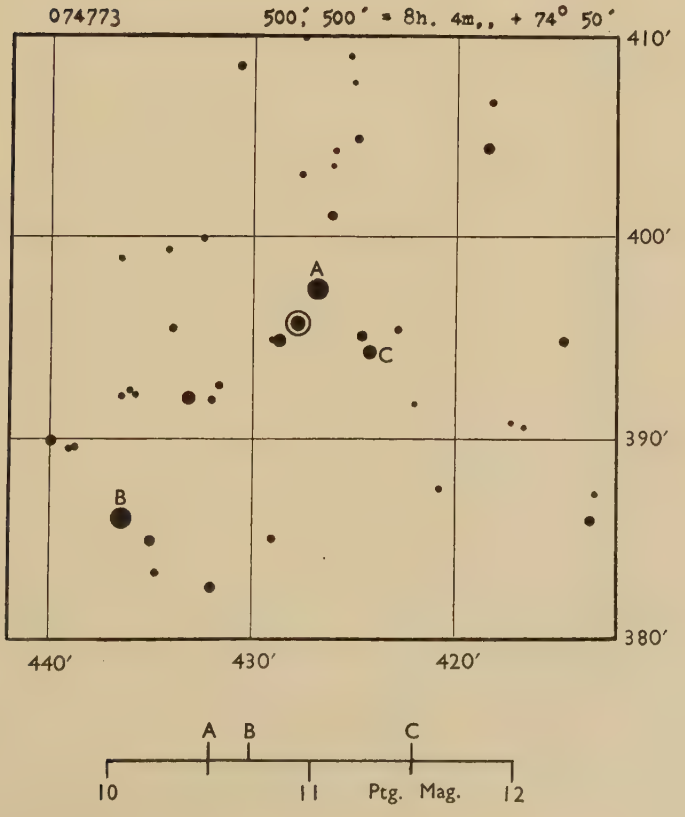

034074
500′, 500′ = 4h. 8m., + 75°
470′
460′
450′
400′
390′
380′
A
B
C
D
E
A B C D E
11
12
Ptg. Mag.
13

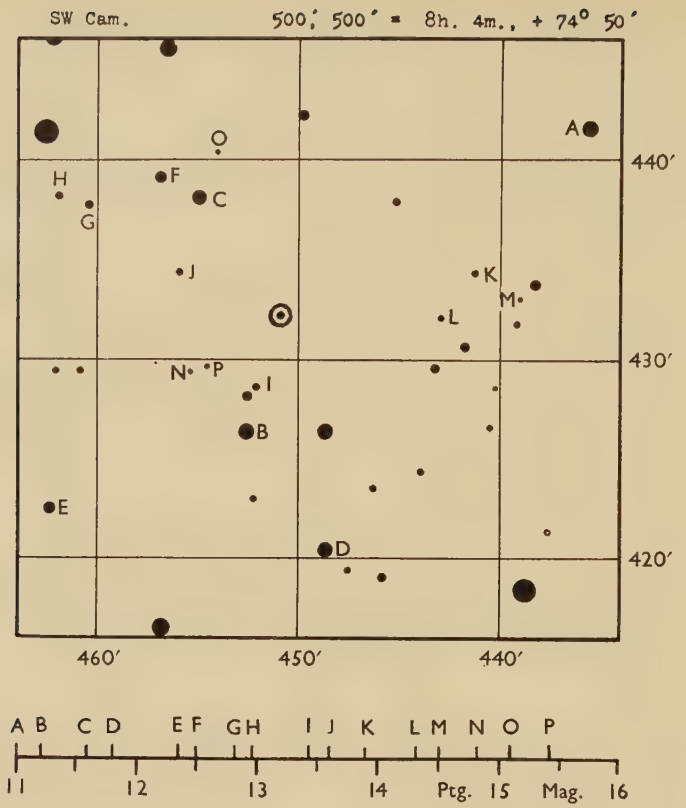

043274 (X Cam.)
500′, 500′ = 4h. 8m., + 75°
510′
500′
490′
610′
600′
590′
A
B
C
D
E
F
G
H
I
J
K
L
A B C D E F G H I J K L
9
10
11
12
13
14
Ptg. 15 Mag.

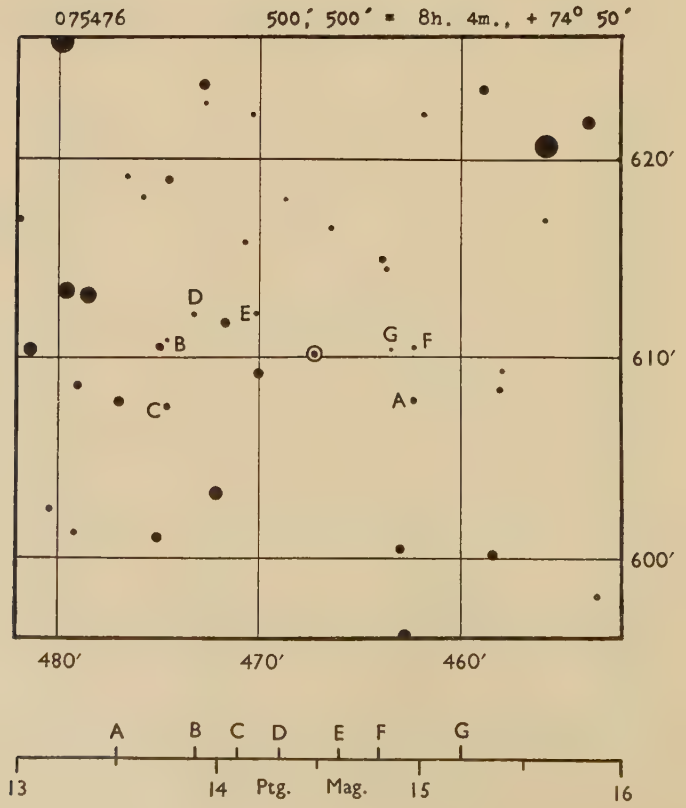

043673
500′, 500′ = 4h.8m., + 75°0′
A
B
C
D
E
F
420′
410′
400′
630′
620′
610′
A B C D E F G
11
12
13
Ptg. Mag.

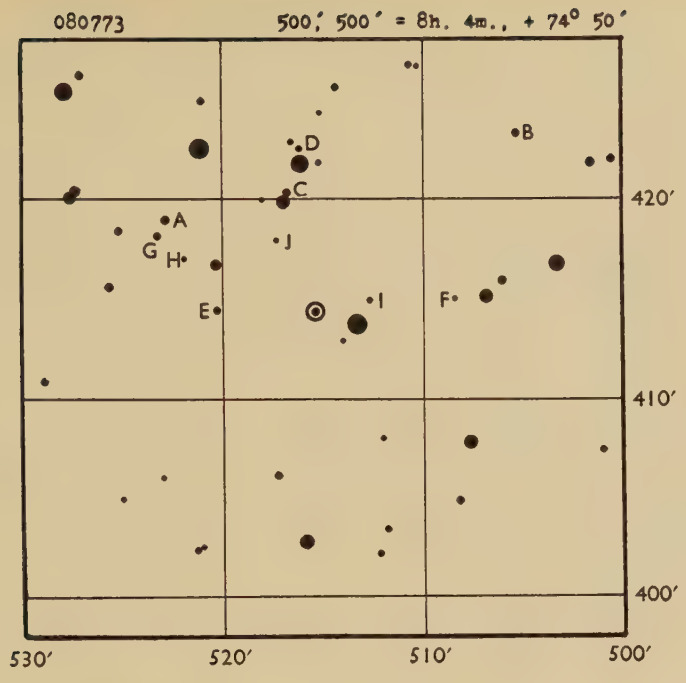

073672
500′, 500′ = 8h.4m., + 74°50′
A
B
C
390′
380′
370′
390′
380′
370′
A B C
13
14
15
Ptg. Mag.

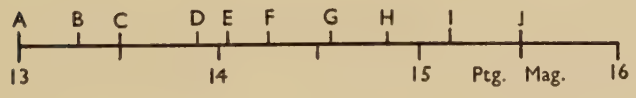

074374
500′, 500′ = 8h. 4m., + 74° 50′
A
B
C
D
E
490′
480′
470′
430′
420′
410′
A B C D E
12
13
Ptg. 14 Mag.

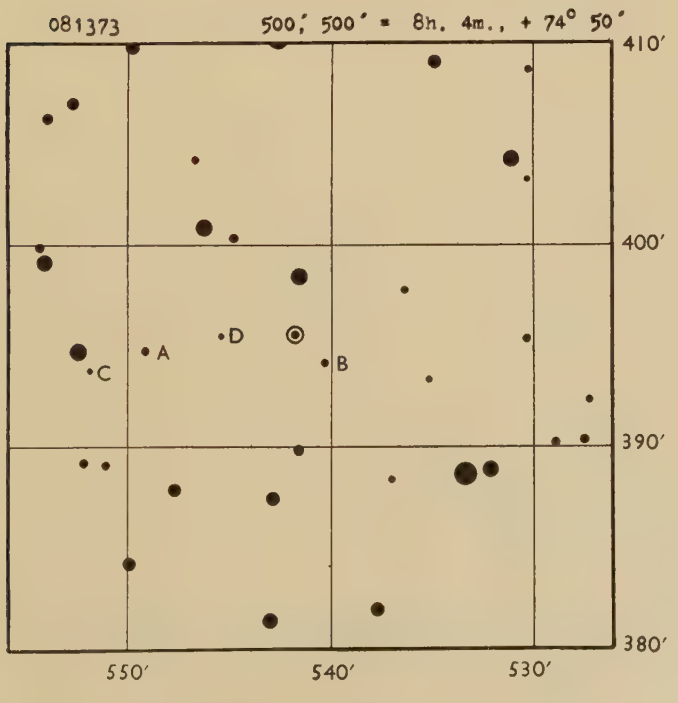

074773
500′, 500′ = 8h. 4m., + 74° 50′
A
B
C
410′
400′
390′
380′
440′
430′
420′
A B C
10
11
Ptg. Mag.
12

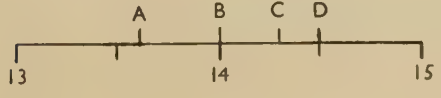

SW Cam.
500′, 500′ = 8h. 4m., + 74° 50′
A
H
F
C
G
J
K
M
L
N
P
I
B
E
D
440′
430′
420′
460′
450′
440′
A B C D E F GH I J K L M N O P
11
12
13
14
Ptg.
15
Mag.
16

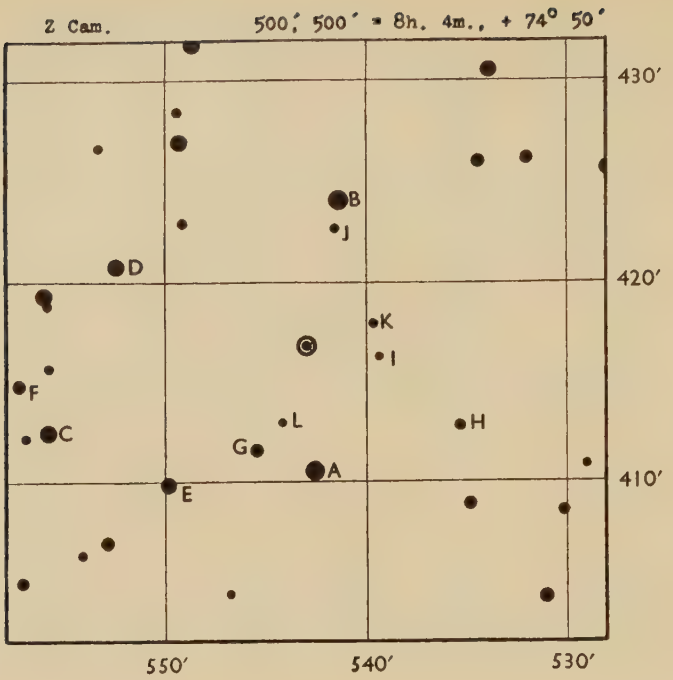

075476
500′, 500′ = 8h. 4m., + 74° 50′
D
E
B
G
F
C
A
620′
610′
600′
480′
470′
460′
A B C D E F G
13
14
Ptg.
Mag.
15
16

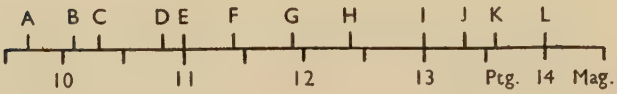

080773
500′, 500′ = 8h. 4m., + 74° 50′
A
B
C
D
E
F
G
H
I
J
420′
410′
400′
530′
520′
510′
500′

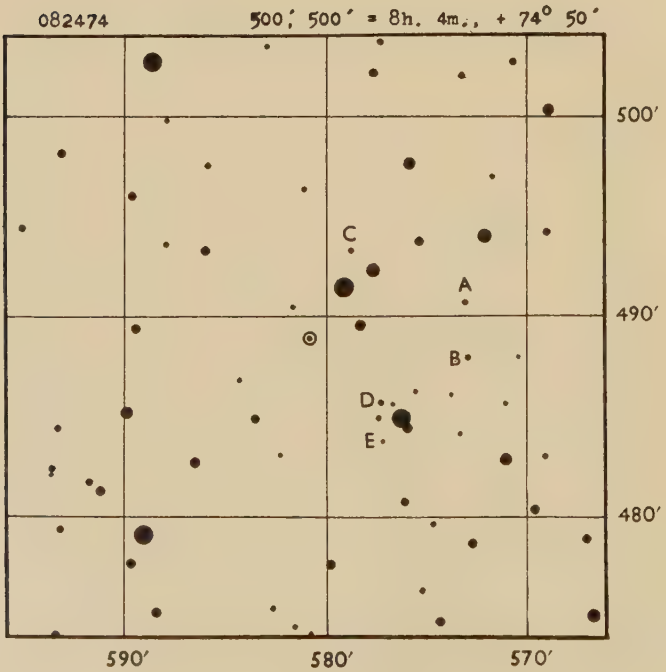

A B C D E F G H I J
13
14
15
Ptg. Mag.
16

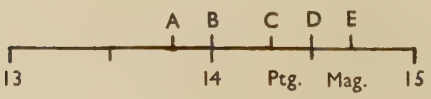

081373
500′, 500′ = 8h. 4m., + 74° 50′
A
B
C
D
410′
400′
390′
380′
550′
540′
530′


A B C D
13
14
15


Z Cam.
500′, 500′ = 8h. 4m., + 74° 50′
430′
420′
410′
550′
540′
530′
B
J
D
K
I
F
C
L
H
G
A
E


A B C D E F G H I J K L
10
11
12
13
Ptg.
14
Mag.


082474
500′, 500′ = 8h. 4m., + 74° 50′
500′
490′
480′
590′
580′
570′
C
A
B
D
E


A B C D E
13
14
Ptg.
Mag.
15

OBSERVATIONS OF COMETS 1937 *b*, *c* AND *f*.

W. H. Steavenson.

The observations which follow (Table I) were made with the 20½-inch Hindle reflector. A cross-bar micrometer was used, in conjunction with a chronograph.

The positions of the comparison stars, given in Table II, include corrections for proper motion where the necessary data were available. The observations of the comet have not been corrected for parallax or differential refraction.

The position of the observatory is :

W. Long. $0^h\ 00^m\ 23^s.10$.
N. Lat. $51^\circ\ 26'\ 09''.9$.

TABLE I

1937 *b* (Whipple)

	U.T.				α (1937·0)			δ (1937·0)			Comet *minus* Star $\Delta\alpha$		Comet *minus* Star $\Delta\delta$		Number of Observations
		d	h	m	h	m	s	°	′	″	m	s	′	″	
1	1937 Feb.	16	22	57·4	13	31	57·35	+39	15	54·9	+0	22·48	+ 9	15·2	10
2		18	0	16·2	13	33	15·56	+39	42	8·5	−2	2·16	+12	6·7	3
3		20	1	48·2	13	35	48·21	+40	34	8·9	−1	59·03	+15	37·6	5
4		23	0	0·8	13	39	19·83	+41	48	24·7	−0	27·92	−11	3·4	12
5		26	1	34·2	13	42	55·85	+43	6	46·8	−1	53·88	+ 0	16·0	7
6	Mar.	12	0	5·9	13	57	39·38	+48	58	18·6	+0	32·51	+ 7	2·9	10
7		18	0	20·9	14	2	58·50	+51	19	55·7	+2	19·41	+ 3	28·6	8
8		18	23	2·4	14	3	44·20	+51	41	16·3	−2	25·75	−12	9·2	2
9		19	22	3·1	14	4	30·30	+52	2	32·9	−1	39·65	+ 9	7·4	10
10		23	0	24·0	14	6	51·40	+53	9	26·9	+1	40·65	+ 6	21·3	6
11		26	22	58·9	14	9	32·41	+54	29	13·4	+0	24·19	−13	5·3	8
12		28	22	7·3	14	10	46·74	+55	6	29·1	−1	10·24	+10	39·0	5
13		29	23	45·6	14	11	24·18	+55	26	9·9	−3	0·54	− 9	14·4	4
14	Apr.	1	0	46·7	14	12	32·32	+56	2	11·8	+1	40·08	+11	10·6	5
15		4	1	16·8	14	14	2·94	+56	51	36·7	+0	5·22	− 7	19·7	10
16		4	21	41·9	14	14	27·55	+57	4	54·3	+0	20·83	+ 4	40·0	10
17		10	1	13·7	14	16	30·30	+58	16	15·3	+2	0·86	− 6	4·2	3
18		11	22	17·8	14	17	9·52	+58	38	42·6	+2	49·02	− 2	7·8	4
19		12	22	15·6	14	17	28·23	+58	49	47·1	+3	7·73	+ 8	56·7	7
20	May	2	21	27·7	14	21	49·20	+60	28	0·9	−0	53·58	− 4	15·7	3
21		3	21	17·4	14	22	1·84	+60	26	39·8	−0	40·94	− 5	36·8	10
22		5	21	51·0	14	22	27·04	+60	21	50·0	−0	15·74	−10	26·6	8
23		10	22	54·7	14	23	41·31	+59	58	54·9	−2	8·08	− 1	20·3	3
24		21	23	1·7	14	27	50·89	+58	14	11·5	−1	25·78	− 2	56·4	5
25		31	23	8·1	14	33	54·04	+55	35	52·1	+1	14·99	−13	47·2	5

TABLE I—*continued*

	U.T.			α (1937·0)			δ (1937·0)			Comet *minus* Star $\Delta\alpha$		Comet *minus* Star $\Delta\delta$		Number of Observations
	d	h	m	h	m	s	°	′	″	m	s	′	″	
26	June 1	22	39·6	14	34	37·32	+55	17	14·0	−0	19·05	+ 9	34·3	5
27	5	0	0·6	14	36	58·39	+54	15	12·0	+0	43·16	− 2	30·0	10
28	5	23	1·3	14	37	46·81	+53	54	42·7	+0	23·25	+12	4·7	10
29	6	22	24·2	14	38	35·80	+53	33	10·3	+1	12·24	− 9	27·7	6
30	8	22	59·5	14	40	22·52	+52	46	51·7	+1	4·10	− 8	4·3	5
31	13	22	39·4	14	45	9·20	+50	42	41·6	+0	44·97	+ 3	4·8	10
32	25	22	56·3	14	58	45·98	+44	48	10·3	+0	57·08	− 1	57·4	8
33	29	23	13·8	15	3	54·07	+42	33	38·5	−1	30·29	− 4	7·7	10
							1937 *c* (Wilk-Peltier)							
34	Mar. 13	20	17·3	1	18	18·55	+40	18	45·4	−1	15·56	+ 1	52·5	1
35	19	19	46·5	1	48	2·10	+50	20	19·8	−0	6·02	− 8	36·5	10
36	Apr. 11	21	37·4	8	32	21·19	+67	41	11·5	+1	36·34	+10	28·1	3
							1937 *f* (Finsler)							
37	July 14	0	51·9	3	17	7·99	+45	40	23·1	+1	6·51	+ 1	36·6	10
38	17	0	48·0	3	22	26·09	+48	40	30·0	−1	7·52	−10	10·9	10
39	29	23	0·2	4	46	30·14	+70	7	1·2	−0	6·66	+10	36·5	4
40	Aug. 2	22	15·3	7	23	47·16	+77	44	12·7	+0	21·40	− 1	26·8	4
41	3	23	23·4	8	39	40·96	+78	6	7·0	−1	8·97	+ 2	17·6	7
42	5	23	48·1	10	54	17·35	+75	3	23·9	−1	28·63	− 4	13·0	4
43	7	23	5·2	12	10	37·13	+68	43	52·9	+0	11·73	+ 4	57·9	10
44	8	22	36·8	12	33	49·93	+64	59	29·9	−0	48·11	+ 4	28·8	7

TABLE II

	B.D. No.	Mag.	Adopted α (1937·0)	Adopted δ (1937·0)	Authority
	°		h m s	° ′ ″	
1	+39 2658	6·2	13 31 34·87	+39 6 39·7	Greenwich 1925 (2).
2	+39 2663	7·2	13 35 17·72	+39 30 1·8	,, ,,
3	+40 2677	8·2	13 37 47·24	+40 18 31·3	,, ,,
4	+42 2431	6·0	13 39 47·75	+41 59 28·1	,, ,,
5	+43 2354	7·8	13 44 49·73	+43 6 30·8	Paris 1875. Bonn A.G. Zone.
6	+49 2259	7·4	13 57 6·87	+48 51 15·7	Greenwich 1925 (2).
7	+51 1889	6·0	14 0 39·09	+51 16 27·1	,, ,,
8	+52 1776	8·3	14 6 9·95	+51 53 25·5	,, ,,
9	+52 1776	8·3	14 6 9·95	+51 53 25·5	,, ,,
10	+53 1689	8·0	14 5 10·75	+53 3 5·6	*Yale Transactions*, vol. iv.
11	+55 1664	8·0	14 9 8·22	+54 42 18·7	Greenwich 1925 (2).
12	+55 1667	7·5	14 11 56·98	+54 55 50·1	,, ,,
13	+55 1669	7·8	14 14 24·72	+55 35 24·3	,, ,,
14	+56 1717	8·9	14 10 52·24	+55 51 1·2	Bonn Veröff., No. 20.
15	+57 1498	6·5	14 13 57·72	+56 58 56·4	Greenwich 1925 (2).
16	+57 1499	6·5	14 14 6·72	+57 0 14·3	,, ,,
17	+58 1491	7·8	14 14 29·44	+58 22 19·5	,, ,,
18	+59 1570	8·4	14 14 20·50	+58 40 50·4	,, ,,
19	+59 1570	8·4	14 14 20·50	+58 40 50·4	,, ,,

TABLE II—*continued*

	B.D. No.	Mag.	Adopted α (1937·0)	Adopted δ (1937·0)	Authority
	°		h m s	° ′ ″	
20	+60 1536	9·2	14 22 42·78	+60 32 16·6	Vatican Astrographic Cat.
21	+60 1536	9·2	14 22 42·78	+60 32 16·6	,, ,,
22	+60 1536	9·2	14 22 42·78	+60 32 16·6	,, ,,
23	+60 1541	7·3	14 25 49·39	+60 0 15·2	Greenwich 1925 (2).
24	+58 1508	8·7	14 29 16·67	+58 17 7·9	Helsingfors Zone, 1875.
25	+56 1749	8·0	14 32 39·05	+55 49 39·3	Greenwich 1925 (2).
26	+55 1699	7·6	14 34 56·37	+55 7 39·7	,, ,,
27	+54 1693	5·8	14 36 15·23	+54 17 42·0	,, ,,
28	+54 1695	7·3	14 37 23·56	+53 42 38·0	,, ,,
29	+54 1695	7·3	14 37 23·56	+53 42 38·0	,, ,,
30	+53 1728	8·2	14 39 18·42	+52 54 56·0	*Yale Transactions*, vol. iv.
31	+51 1951	7·0	14 44 24·23	+50 39 36·8	Greenwich 1925 (2).
32	+45 2244	6·8	14 57 48·90	+44 50 7·7	,, ,,
33	+42 2566	8·0	15 5 24·36	+42 37 46·2	,, ,,
34	+39 315	7·7	1 19 34·11	+40 16 52·9	,, ,,
35	+50 379	6·1	1 48 8·12	+50 28 56·3	,, ,,
36	+67 552	9·3	8 30 44·85	+67 30 43·4	Pub. Cincinnati Obs., No. 20.
37	+45 740	7·4	3 16 1·48	+45 38 46·5	Greenwich 1925 (2).
38	+48 920	6·0	3 23 33·61	+48 50 40·9	,, ,,
39	+69 280	9·0	4 46 36·80	+69 56 24·7	Greenwich Astr. Cat.
40	+77 288	9·0	7 23 25·76	+77 45 39·5	,, ,,
41	+78 292	8·6	8 40 49·93	+78 3 49·4	,, ,,
42	+75 431	8·0	10 55 45·98	+75 7 36·9	,, ,,
43	+69 652	9·5	12 10 25·40	+68 38 55·0	,, ,,
44	+65 891	8·6	12 34 38·04	+64 55 1·1	,, ,,

STELLAR PARALLAXES DETERMINED AT THE ROYAL OBSERVATORY, GREENWICH.

(Communicated by the Astronomer Royal)

The following list is a continuation of the relative parallaxes obtained with the 26-inch refractor, and published annually in the November number of the *Monthly Notices*.

The visual magnitude and spectral type for stars contained in the *Draper Catalogue* have been extracted from that catalogue. The photographic magnitudes are from the *Greenwich Astrographic Catalogue*.

Two values of the proper motion in R.A. are given. The first (as also the proper motion in Dec.) is absolute, and is based on meridian observations or the comparison of long-interval plates. The second is the relative proper motion derived from the parallax plates.

In the last column the first value of the probable error refers to the parallax, and the second to the relative proper motion in R.A. These are based on the value of the probable error for a plate of unit weight given in the eighth column. The mean probable error for unit weight is $\pm$ 0″·025.

To minimise the effect of *quasi*-seasonal error the observations have been spread over not less than six epochs. The average number of plates per star is 29; for those stars observed during the summer months the number is in excess of the average to compensate for the loss of factor.

Generally six comparison stars of mean magnitude 11–12 have been used, the magnitude of the parallax star being reduced to equality by means of a rotating sector. For the stars in the present list no supplementary reduction has been necessary. Two exposures of 3 minutes each are made on each plate, the carrier being turned through 180° after the first exposure.

The present list includes three faint stars (Nos. 2, 3 and 7) which are designated by their numbers in the *Greenwich Astrographic Catalogue*. The first of these, A.C. 71° 532, has a relative parallax of +0″·114 and a measured proper motion of 178 seconds of arc per century. This star appears to be moving nearly parallel to B.D. 66° 34 whose measured proper motion is 176 seconds of arc per century and whose relative parallax has been measured at Greenwich and comes out as +0″·093.* The stars are 5°·1 apart in the sky.

* No. 270 in *Greenwich Observations of Stellar Parallax*, Vol. II.

No.	Name and B.D. No.	Mag. Vis. Phot.	Type	R.A. Dec. 1900·0	Proper Motion (Unit 0″·001) R.A.	Proper Motion (Unit 0″·001) Dec.	P.E. Unit Weight	Parallax.	P.E.
				h m s					
1	L. 1133	6·9	F8	0 38·40	+ 212	−059	22	+ 15	± 6
	66° 58	7·2		+66° 37′	+ 215				± 2
2	A.C. 71° 532	...		0 55 7	+1735	−395	31	+114	± 8
		10·5		+71° 9′	+1749				± 2
3	A.C. 75° 1146	...		2 57 32	+ 219	− 50	24	+ 48	± 6
		10·8		+75° 40′	+ 175				± 4
4	Gr. 666	7·9	Ko	3 22 39	+ 4	− 62	22	− 6	± 5
	66° 269	8·6		+67° 5′	− 15				± 4
5	O.A. 4884	9·2	Ko	4 26 45	+ 55	− 45	28	+ 3	± 7
	67° 338	9·8		+67° 8′	+ 19				± 5
6	P. IV 180	8·2	Ko	4 44 30	− 8	− 69	20	0	± 5
	66° 360	8·3		+66° 16′	− 29				± 3
7	A.C. 68° 2911	...		6 53 45	+ 335	+ 69	31	+ 54	± 8
		12·3		+68° 25′	+ 318				± 5
8	L. 13875	7·6	F8	7 9 7	− 66	−138	22	+ 8	± 6
	66° 493	7·8		+66° 55′	− 93				± 4
9	L.F. 1411	7·0	A5	8 57 15	+ 11	− 56	24	+ 8	± 7
	69° 504	7·1		+68° 50′	+ 21				± 3
10	P. IX 91	7·2	F5	9 27 54	− 122	− 89	21	+ 12	± 5
	72° 464	7·5		+72° 32′	− 90				± 3
11	69° 535	9·6		9 39 12	− 4	+ 22	24	− 12	± 7
		9·7		+68° 57′	− 1				± 3
12	L.F. 1562	7·5	Ko	9 46 4	− 57	− 17	24	− 5	± 7
	78° 327	8·4		+78° 25′	− 82				± 3
13	O.A. 10437	8·7	F8	9 56 12	− 35	− 65	23	+ 18	± 6
	67° 631	9·2		+66° 55′	− 40				± 3
14	L.F. 1653	6·6	A3	10 9 48	− 32	− 49	29	+ 23	± 8
	71° 534	6·9		+71° 34′	− 17				± 4
15	L.F. 1654	7·2	A3	10 9 48	− 20	− 35	25	+ 20	± 7
	71° 534	7·5		+71° 33′	− 9				± 4
16	O.A. 11222	8·1	F8	10 49 16	+ 148	+ 51	20	+ 18	± 5
	66° 689	8·6		+66° 16′	+ 123				± 3
17	Gr. 1747	7·1	Ko	11 8 48	− 72	− 34	28	+ 9	± 7
	79° 356	7·9		+78° 51′	− 53				± 4

No.	Name and B.D. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Unit 0″·001 Proper Motion R.A.	Dec.	P.E. Unit Weight.	Parallax	P.E.
				h m s					
18	L.F. 1879	7·3	Ko	11 23 15	− 52	+ 23	24	− 11	± 6
	68° 644	8·0		+68° 1′	− 42				± 3
19	L.F. 2088	7·0	Ko	12 26 38	− 64	− 20	23	0	± 6
	72° 569	7·7		+71° 51′	− 52				± 4
20	O.A. 14066–7	7·7	Ko	13 48 29	+ 43	− 49	22	− 3	± 7
	69° 723	8·4		+69° 39′	+ 22				± 4
21	Gr. 2087	6·6	Ko	14 8 8	+ 10	− 7	29	− 6	± 8
	69° 736	7·5		+69° 20′	− 21				± 4
22	O.A. 14736	9·2	Ko	14 32 52	+ 18	− 64	24	− 2	± 7
	69° 757	9·2		+69° 8′	+ 8				± 4
23	Gr. 2210	7·0	Ko	14 49 38	− 21	− 5	30	− 21	± 9
	86° 217	7·7		+86° 22′	+ 10				± 4
24	L.F. 2690	7·0	Ko	15 35 57	+ 16	+ 8	28	− 30	± 8
	81° 517	7·6		+81° 6′	+ 22				± 3
25	67° 941	7·4	Ko	16 25 4	+ 17	− 6	24	− 3	± 9
		8·0		+67° 18′	− 2				± 3
26	O.A. 19194	8·9	Ko	19 17 45	+ 41	+ 28	30	+ 6	± 11
	65° 1340	9·4		+65° 10′	+ 36				± 3
27	O.A. 19326	9·1	Ko	19 26 11	+ 29	+ 63	28	− 19	± 9
	74° 823	9·4		+74° 49′	+ 39				± 3
28	C. 3638	10·0		23 35 57	+ 160	+ 77	27	+ 18	± 9
	85° 404	10·5		+86° 7′	+ 169				± 2

Nos. 14, 15. Σ 1415.
No. 23. Σ 1915. Brighter component measured.

STELLAR PARALLAXES DETERMINED PHOTOGRAPHICALLY AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE. (NINTH LIST.)

(Communicated by H.M. Astronomer)

The form of publication is similar to that of previous lists.

Sirius and Canopus, the two brightest stars in the sky, are included. For both these stars there is a very good field for the selection of reference stars and for each of them two sets of reference stars were selected, of magnitudes 11·0 and 11·7 approximately. In the case of Sirius the reduction of brightness was secured by means of a filter cutting down the brightness by about 11 magnitudes in conjunction with a rotating shutter reducing the brightness by an additional 0·7 or 1·5 magnitudes. For Canopus the same filter was used with the 0·7 shutter for one series of plates and a filter cutting down by about $5\frac{1}{2}$ magnitudes in combination with a sector reducing by 5 magnitudes. For each of the stars there are therefore two independent determinations. Unfortunately both these stars have to be observed in the summer months when the definition is frequently poor, but additional accuracy has been secured by a larger number of plates extending over seven epochs of maximum parallactic displacement. The number of plates used for the two determinations of Sirius was 41, 45, and of Canopus 42, 44.

In the case of Sirius orbital motion was allowed for by means of Volet's orbit given in *Bull. Astr.*, 7, 30, assuming the ratio of the mass of the faint component to the brighter as 0·47. The remaining residuals in both solutions show a systematic run from epoch to epoch. The probable error for a plate of unit weight and the systematic run of the residuals is actually larger than is the case when orbital motion is neglected. During this series of observations the orbital motion in R.A. amounted to about $+0''{\cdot}25$ annually.

The parallax of Canopus comes out considerably larger than that generally adopted. If the parallax of this star is $''{\cdot}025$ instead of $''{\cdot}005$ generally adopted, then its brightness is 3000 and not 80,000 times that of the Sun. Further observations will be made.

Another very bright star in the list is Antares. This is a giant M-type star, and the parallax determined is smaller than that obtained by trigonometric methods at other observatories, but agrees more closely with the spectroscopic determination and that deduced for it as a member of the Scorpio-Centaurus cluster. The very large linear diameter of this star is based on the latter parallax. It should be mentioned that this is a very difficult star for a parallax determination on account of the difficulty in finding suitable reference stars and the presence of a faint companion separated from the principal star by only $3''$ in R.A.

The present list contains a number of stars of very small parallax of type M and without proper motion. These stars were observed merely

on account of their spectral type, to fill up gaps in the observing programme.

Most of the proper motion determinations used in making up the observing list have been reasonably confirmed. The star No. 808 was observed for the star given by Innes as U.O.C. 58, 40, but Luyten has already reported that this star is spurious. The actual star observed has been identified in the Astrographic Catalogue.

Three stars for which parallaxes had previously been published are included in this list with the original number. As the original material appeared rather weak, some additional plates were secured to increase the weight of the determination. The list contains seven stars with a parallax exceeding 0″·100, including Barnard's star and several other well-known stars of large parallax. Star No. 774 is new, and the parallax is much larger than would be anticipated from the proper motion.

For the fainter stars the positions given have been checked from the plates. The letters Z.C. and F.S. refer to the Cape Zone Catalogue for 1900 and the unpublished catalogue of faint stars in the Cape Astrographic Zone. Attention is being given more and more to stars in the Cape Zone $-40°$ to $-52°$, although the working list still contains many very faint stars with large proper motion discovered by Wolf and Ross. A few stars in this list are double stars with dynamical parallaxes.

No.	Name and D.M. No.	Mag. Vis. Ptg.	Spec. Type	R.A., Dec. (1900)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
501	Ci. 18, 5	8·8	G5	0 2 24	+ 322	+ 85	25	+ 36	± 8
	−24° 8	9·3		−24° 22′·7	+ 324				± 4
751	U.O.C. 48	9·0	Go	0 6 31	+ 395	+ 73	26	+ 47	± 9
	−59° 8	8·8		−59° 27′·9	+ 273				± 9
752	U.O.C. 48	9·4	...	0 18 2	...	−570	27	+ 21	± 9
	−53° 77	10·2		−53° 4′·0	+ 55				± 9
753	β Hydri	2·90	Go	0 20 30	+2220	+323	20	+168	± 6
	Boss 74	3·46		−77° 49′·0	+2177				± 7
754	U.O.C. 53	...	...	2 4 40	+1759	+384	34	+ 62	± 12
	Ci. 20, 149	12·5		−67° 3′·2	+1739				± 10
755	−36° 874	8·2	G5	2 15 35	− 85	0	23	+ 28	± 9
	h 3494	9·0		−35° 54′·3	− 67				± 9
756	Boss 575	4·82	F5	2 27 21	− 79	−117	19	+ 11	± 6
	σ Ceti	5·24		−15° 41′·0	− 100				± 5
757	U.O.C. 39	...	...	3 37 12	+ 192	−320	26	+ 22	± 9
	F.S. 1054	12·3		−45° 42′·6	+ 172				± 8
758	Boss 884	5·42	Ao	3 44 54	+ 77	− 15	24	+ 18	± 7
	f Eridani B	5·42		−37° 55′·7	+ 20				± 5

No.	Name and D.M. No.	Mag. Vis. Ptg.	Spec. Type	R.A., Dec. (1900)	Unit 0″·001 Proper Motion R.A.	Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
759	Boss 885	4·86	B8	3 44 54	+ 80	− 18	26	+ 24	± 8
	f Eridani A	4·81		−37° 55′·5	+ 52				± 5
	Mean				+ 79	− 16	25	+ 21	± 5
	Boss 884, 885				+ 36				± 4
760	Z.C. 2141	9·1	G5	4 12 40	+ 241	−130	20	− 18	± 6
	−46° 1355	9·5		−46° 12′·9	+ 202				± 6
761	F.S. 1313	...	...	4 24 1	+ 188	−174	28	− 1	± 9
	−41° 1420	11·9		−41° 13′·7	+ 196				± 8
762	−59° 370	6·38	Go	4 38 38	+ 78	+183	34	+ 61	± 11
	h 3683	7·1		−59° 8′·3	+ 9				± 6
763	F.S. 1422	...	Go	4 39 17	− 225	− 15	29	− 11	± 12
	−44° 1666	10·5		−44° 32′·2	− 299				± 9
764	ζ Doradus	4·76	F8	5 3 48	− 43	+105	27	+108	± 7
	Boss 1225	6·2		−57° 36′·6	− 58				± 6
765	η Leporis	3·77	Fo	5 51 51	− 41	+132	21	+ 91	± 7
	Boss 1476	4·05		−14° 11′·2	− 55				± 6
766	U.O.C. 53	...	...	6 10 41	+ 214	−779	22	+ 55	± 8
	Ci. 20, 381	12·3		−65° 10′·0	+ 176				± 8
*767	Canopus (1)	−·86	Fo	6 21 44	+ 16	+ 10	26	+ 25	± 7
	α Carinæ	−·58		−52° 38′·5	+ 7				± 5
	Canopus (2)						20	+ 28	± 6
					+ 8				± 4
	Mean							+ 27	± 4
768	F.S. 2339	...	...	6 34 19	+ 362	−232	28	− 1	± 9
	−40° 2586	11·6		−40° 58′·6	+ 370				± 9
769	F.S. 2404	...	Go	6 40 37	+ 14	+291	27	+ 15	± 9
	−48° 2470	11·1		−48° 34′·5	+ 69				± 9
*770	Sirius (1)	−1·58	Ao	6 40 45	− 539	−1206	31	+415	± 9
	α Canis Maj.	−1·58		−16° 34′·7	− 552				± 6
	Sirius (2)				− 539	−1206	33	+380	± 10
					− 581				± 6
	Mean							+397	± 7

* For Canopus and Sirius, (1) refers to the series of plates with short exposure, (2) to the series with longer exposure.

No.	Name and D.M. No.	Mag. Vis. Ptg.	Spec. Type	R.A., Dec. (1900)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
771	F.S. 2504	...	Ko	6 47 58	+ 32	+190	27	− 9	± 10
	−50° 2419	10·9		−50° 55′·9	+ 28				± 10
772	−2° 1899	7·9	A3	6 58 25	− 379	+ 45	22	+ 31	± 7
		8·0		−2° 52′·7	+ 14				± 7
773	Ci. 18, 917	9·3	...	7 34 45	+ 153	−240	32	+ 37	± 9
	−1° 1792	10·3		−1° 17′·2	+ 170				± 10
774	Ci. 18, 918	7·25	K2	7 34 59	+ 85	−255	27	+114	± 8
	−3° 2001	8·32		−3° 21′·6	+ 123				± 8
775	Ci. 18, 948	8·45	Ko	7 53 50	+ 367	−266	21	+ 41	± 6
	−25° 5342	9·2		−25° 21′·0	+ 365				± 6
776	Boss 2227	5·67	F5	8 19 36	− 214	− 29	23	+ 48	± 7
	1 Hydræ	6·09		−3° 25′·6	− 190				± 5
777	U.O.C. 58, 15	...	...	8 56 11	− 528	+676	35	+ 71	± 11
	Ci. 20, 506	14·0		−47° 3′·5	− 478				± 6
778	s.f. prec. star	...	...	8 56 17	...	...	29	+ 5	± 9
		14·0		−47° 4′·7	+ 32				± 5
441	Ci. 20, 522	...	...	9 19 12	− 825	+140	31	+137	± 10
	−59° 1362	10·5		−59° 51′·5	− 842				± 3
779	F.S. 6066	...	K2	9 21 28	+ 190	− 68	20	+ 19	± 6
	−48° 4673	11·0		−48° 47′·5	+ 185				± 6
780	U.O.C. 37	...	...	10 8 18	− 229	+313	23	+ 21	± 7
	−70° 982	11·0		−70° 57′·5	− 277				± 7
781	Ci. 18, 1264	8·1	Ko	10 23 13	− 394	−271	23	+ 47	± 7
	−5° 3071	9·1		−6° 4′·9	− 392				± 8
782	−8° 2963	7·12	Go	10 32 1	− 111	− 44	29	+ 18	± 10
	A.D.S. 7854	7·68		−8° 19′·2	− 54				± 10
783	ν Hydræ	3·32	Ko	10 44 41	+ 94	+193	33	+ 35	± 10
	Boss 2888	4·32		−15° 40′·2	+ 51				± 11
784	F.S. 8654	...	G5	11 4 57	+ 280	−274	32	+ 38	± 9
	−41° 6367	10·6		−41° 55′·1	+ 288				± 9
785	Ci. 18, 1491	6·90	Go	11 56 41	− 238	0	25	+ 43	± 9
	−33° 8130	7·6		−34° 5′·6	− 145				± 9
786	ϵ Corvi	3·21	Ko	12 4 59	− 65	+ 7	31	− 9	± 9
	Boss 3172	4·21		−22° 3′·8	− 78				± 8

No.	Name and D.M. No.	Mag. Vis. Ptg.	Spec. Type	R.A., Dec. (1900)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
787	U.O.C. 48	...	...	12 27 8	...	−339	25	− 14	± 8
	−56° 5292	10·5		−56° 13′·3	+ 4				± 6
788	−86° 283	7·10	Go	12 57 24	− 232	−146	25	− 8	± 9
	Cp. A. XI, p. 397	7·9		−87° 1′·4	− 216				± 8
789	U.O.C. 48	8·3	Go	13 9 17	− 308	−137	27	+ 14	± 8
	−54° 5505	9·0		−54° 48′·5	− 201				± 8
790	γ Hydræ	3·33	G5	13 13 29	+ 67	− 51	23	+ 15	± 8
	Boss 3449	4·11		−22° 38′·6	+ 72				± 6
791	Alb. 4597	7·12	F5	13 45 57	− 329	− 53	22	+ 16	± 6
	−30° 10960	8·0		−30° 49′·3	− 336				± 7
792	Ci. 20, 833	...	...	14 2 54	− 435	−599	22	+ 53	± 6
	−60° 5215	11·6		−61° 1′·6	− 480				± 5
793	U.O.C. 48	...	...	14 12 50	− 255	−159	26	+ 21	± 8
		11·5		−62° 36′·5	− 215				± 8
794	Ci. 18, 1884	8·2	Go	14 17 38	− 266	−279	22	+ 35	± 6
	−27° 9806	8·3		−27° 21′·6	− 213				± 8
795	F.S. 12148	...	K	14 23 48	− 647	−402	19	+ 21	± 6
	−46° 9361	11·4		−46° 29′·6	− 651				± 7
796	Ci. 18, 1914	6·09	F5	14 29 23	− 360	−256	24	+ 42	± 8
	−67° 2616	7·6		−67° 29′·3	− 368				± 8
797	U.O.C. 48	...	...	14 32 38	+ 289	−372	33	+ 23	± 10
	Ci. 20, 874	12·5		−56° 28′·4	+ 377				± 10
798	α Circini	3·41	Fo	14 34 25	− 200	−236	27	+ 49	± 8
	Boss 3739	3·69		−64° 32′·4	− 161				± 10
799	Ci. 20, 919	6·66	Ko	15 8 50	−1279	−506	24	+ 51	± 7
	−0° 2944	7·66		−0° 57′·8	−1325				± 8
800	Z.C. 13429	7·5	G5	15 22 10	− 238	− 91	30	+ 54	± 8
	−49° 9653	8·6		−49° 36′·2	− 198				± 10
801	Ci. 20, 935	...	...	15 26 34	− 456	+ 40	23	+ 3	± 8
	Ross 805	12·0		−11° 1′·9	− 366				± 9
802	Alb. 5391	7·8	Go	15 57 37	− 323	−270	26	+ 12	± 8
	−36° 10618	8·0		−37° 1′·8	− 309				± 8
803	Ci. 18, 2143	8·9	Ko	15 58 11	− 275	−164	26	+ 51	± 8
	−21° 4264	8·9		−21° 38′·9	− 334				± 8

No.	Name and D.M. No.	Mag. Vis. Ptg.	Spec. Type	R.A., Dec. (1900)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
804	U.O.C. 58, 34	...	...	16 15 21	− 617	−465	23	+ 33	± 7
		13·5		−48° 24′·5	− 589				± 6
805	Antares	1·22	Ma	16 23 17	− 7	− 33	24	+ 6	± 7
	α Scorpii	2·57	A3	−26° 12′·6	− 53				± 7
806	Alb. 5731	8·3	K5	16 51 58	+ 282	+194	23	+ 71	± 6
	−39° 10940	9·2		−39° 24′·6	+ 259				± 9
807	Boss 4413	5·96	Ko	17 18 43	− 26	− 36	25	+ 9	± 6
	−21° 4597	7·4		−21° 20′·9	− 31				± 9
808	C.A.Z.	...	...	17 47 19	...	...	30	+ 13	± 9
	−42°, 108, 304	13		−41° 56′·5	− 10				± 10
809	Barnard's star	9·7	Mb	17 52 57	− 720	+10210	26	+526	± 8
	+4° 3561	11·2		+4° 30′	− 725				± 8
810	−27° 12647	8·7	Ma	18 11 23	...	...	21	− 9	± 6
		10·0		−27° 55′·9	+ 4				± 8
811	−21° 5013	8·9	Ma	18 22 11	...	...	23	− 24	± 7
		10·5		−21° 18′·0	− 6				± 9
812	U.O.C. 53	8·05	G5	18 52 56	− 281	−289	24	+ 25	± 7
	−65° 3763	8·2		−65° 3′·4	− 256				± 9
813	Ci. 18, 2497	8·7	Go	19 3 43	− 261	−389	21	+ 6	± 6
	−21° 5273	8·9		−21° 37′·1	− 284				± 7
814	π Sagittarii	3·02	F2	19 3 49	− 6	− 40	21	+ 18	± 6
	Boss 4874	3·36		−21° 11′·0	− 34				± 7
815	Z.C. 18597	9·5	G5	19 59 22	− 36	−250	29	− 12	± 8
	−43° 13796	9·8		−43° 29′·6	− 67				± 7
816	Ci. 18, 2674	7·10	Go	20 34 27	+ 175	−304	29	+ 7	± 9
	−24° 16195	7·9		−24° 27′·8	+ 196				± 12
817	U.O.C. 58, 45	...	...	21 5 13	+ 176	−656	26	+ 50	± 9
	−44° 14334	13·0		−44° 0′·6	+ 205				± 9
818	Ci. 20, 1269	...	...	21 7 57	− 183	−1060	32	+ 11	± 9
	Ross 770	12·6		−19° 42′·7	− 232				± 7
819	Ci. 18, 2799	8·5	Ko	21 26 47	− 299	−150	26	+ 24	± 8
	−21° 6035	8·7		−21° 23′·7	− 309				± 11
820	Ci. 18, 2801	8·1	G5	21 28 13	− 576	−110	23	+ 28	± 6
	−50° 13411	8·5		−50° 14′·0	− 582				± 7

No.	Name and D.M. No.	Mag. Vis. Ptg.	Spec. Type	R.A., Dec. (1900)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
821	Ci. 20, 1298	8·8	G5	21 33 1	− 502	−262	30	+ 36	± 10
	−2° 5588	9·6		−2° 44′·7	− 502				± 11
822	δ Capric	2·98	A5	21 41 31	+ 258	−295	27	+ 59	± 9
	Boss 5600	3·12		−16° 34′·9	+ 212				± 9
823	Ci. 20, 1315	...	...	21 44 25	− 205	−480	24	+ 32	± 8
	Ross 209	11·8		−12° 7′·8	− 338				± 9
824	Ci. 20, 1337	...	...	22 4 2	+ 59	−710	27	+ 45	± 10
	Wolf 1328	14·3		−8° 23′·0	− 35				± 10
825	Boss 5725	6·40	Go	22 8 32	+ 546	−782	26	+ 40	± 8
	−41° 14804	6·4		−41° 51′·3	+ 567				± 9
826	Ci. 18, 2920	7·76	K2	22 23 35	+ 207	−840	30	+ 81	± 9
	−30° 19175	8·2		−30° 30′·4	− 253				± 11
827	Ci. 18, 2929	8·2	G5	22 26 27	+ 293	−170	21	− 10	± 6
	−31° 18861	9·0		−31° 41′·0	+ 278				± 7
589	Ci. 20, 1387	...	...	22 47 51	+ 972	−560	26	+218	± 10
	−15° 6290	11·5		−14° 46′·7	+ 976				± 5
828	Ci. 20, 1391	8·9	G5	22 50 33	+ 564	− 70	28	+ 20	± 9
	−8° 5980	9·7		−8° 21′·3	+ 498				± 10
829	Alb. 7982	8·8		22 52 21	+ 198	−371	23	− 11	± 7
	−26° 16395*p*			−26° 38′·0	+ 129				± 11
830	Alb. 7983	7·6		22 52 21	+ 155	−327	31	+ 12	± 9
	−26° 16395*s*			−26° 38′·1	+ 142				± 8
	Mean	7·27	Go					0	± 6
		7·6			+ 136				± 7
831	Ci. 20, 1403	8·3	K2	23 3 58	+ 581	− 87	24	+ 7	± 7
	−3° 5577	9·4		−2° 48′·0	+ 609				± 9
832	Ci. 18, 3036	6·24	Go	23 7 57	+ 460	−420	23	+ 37	± 7
	−63° 4862	7·2		−63° 13′·9	+ 304				± 7
833	Ci. 18, 3048	8·3	F8	23 11 55	− 488	−1206	23	+ 3	± 8
	−14° 6437	8·8		−14° 21′·8	− 505				± 9
834	−13° 6407	7·50	Ma	23 22 14	...	...	31	+ 16	± 11
		8·85		−13° 28′·8	− 2				± 14
835	Ci. 20, 1434	...	...	23 29 30	+ 393	−100	24	+ 11	± 7
	−35° 15842	10·8		−35° 13′·2	+ 481				± 7
836	Ci. 20, 1459	9·4	Ko	23 46 52	+ 202	−753	25	+ 9	± 8
	−62° 6446	10·4		−61° 57′·5	+ 133				± 10

MICROMETRICAL MEASURES OF DOUBLE STARS.

Rev. T. E. R. Phillips, M.A.

The following measures have been made with the 8-inch refractor lent by the Society. Special attention has again been given to Σ 1523.

Star	1925 R.A.	1925 Dec.	Mags.	Position Angle	Distance	No. of Nights	Date 1900 +
	h m	° ′		°	″		
AC 1	0 16·9	+32 34	6·5, 7·0	286·0	1·57	3	36·68
Σ 60 (η Cas)	0 44·4	+57 26	4·0, 7·6	276·5	8·67	5	35·74
				278·2	8·91	6	37·72
Σ 73 (36 And)	0 50·9	+23 14	6·2, 6·8	105·3	0·58	7	37·00
Σ 113 (42 Cet)	1 16·0	− 0 54	6·2, 7·2	1·1	1·41	3	35·80
Σ 162	1 44·5	+47 32	7·0, 7·5	209·5	1·93	3	37·79
Σ 174 (1 Ari)	1 46·0	+21 54	6·2, 7·4	165·8	2·76	3	37·74
Σ 178	1 48·0	+10 26	7·8, 7·8	199·2	2·98	3	36·79
Σ 186	1 52·0	+ 1 28	7·2, 7·2	42·9	1·40	4	35·00
				43·9	1·42	3	36·90
Σ 202 (α Psc)	1 58·2	+ 2 24	2·8, 3·9	306·1	2·54	3	35·81
OΣ 38 (γ^2 And)	1 59·3	+41 58	5·0, 6·2	105·5	0·48	3	36·66
				102·8	0·44[a]	6	37·50
Σ 285	2 34·4	+33 6	7·0, 7·7	165·6	1·60	3	37·84
Σ 299 (γ Cet)	2 39·4	+ 2 56	3·7, 6·2	292·8	3·00	3	35·84
Σ 333 (ϵ Ari)	2 54·9	+21 3	5·7, 6·0	204·4	1·54	3	36·80
Σ 425	3 35·4	+33 53	7·3, 7·3	83·9	2·19	3	37·81
Σ 572	4 33·9	+26 48	6·5, 6·5	197·8	4·06	3	37·81
OΣ 95	5 1·1	+19 42	7·0, 7·6	315·2	0·81	2	36·86
OΣ 98 (14 Ori)	5 3·8	+ 8 24	5·9, 6·7	109·5	0·90	4	37·82
Σ 644	5 5·2	+37 13	6·7, 7·0	321·0	1·61	4	37·80
OΣ 545 (θ Aur)	5 54·6	+37 13	3·0, 7·5	326·1	2·90	3	37·81
Σ 1110 (Castor)	7 29·8	+32 3	2·0, 2·8	206·2	4·04	5	34·08[b]
				205·5	3·91	4	35·05
				205·3	3·94	5	36·06
				204·4	3·89	5	37·17
OΣ 175	7 30·4	+31 7	6·0, 6·6	328·7	0·65	5	36·17
Σ 1126	7 36·1	+ 5 24	7·2, 7·5	151·5	1·09	6	37·25
Σ 1196 (ζ Cnc)	8 7·9	+17 53	5·0, 5·7	124·0	0·73	1	35·18
				117·4	0·71	11	36·18
				108·2	0·71	7	37·21
Σ 1291	8 49·7	+30 52	5·9, 6·4	319·7	1·36	3	37·31
Σ 3121	9 13·5	+28 54	7·5, 7·8	22·8	0·67	4	37·29
Σ 1356 (ω Leo)	9 24·4	+ 9 23	5·9, 6·7	141·0	0·91	3	36·29
OΣ 215	10 12·2	+18 6	7·2, 7·4	194·2	1·11	3	37·35

Star	1925 R.A. h m	1925 Dec. ° ′	Mags.	Position Angle °	Distance ″	No. of Nights	Date 1900 +
Σ 1523 (ξ UMa)	11 14·2	+31 58	4·0, 4·9	316·3	1·26	4	35·12
				311·3	1·14	3	35·43
Annual Means	° 311·8	″ 1·13	13 1935·39	309·1	1·07	3	35·50
				308·9	1·03	3	35·62
				305·1	1·41	4	36·10
	302·3	1·40	13 1936·40	302·3	1·41	5	36·31
				299·6	1·37	4	36·60
				294·6	1·52	4	37·13
	292·2	1·54	15 1937·41	291·7	1·51	5	37·42
				290·7	1·57	6	37·63
Σ 1639	12 20·7	+26 0	6·7, 7·9	332·3	0·91	4	37·47
Σ 1670 (γ Vir)	12 37·9	− 1 2	3·6, 3·7	318·7	5·94	3	36·20
Σ 1687 (35 Com)	12 49·6	+21 39	5·0, 7·8	117·6	0·95	7	37·24
Σ 1834	14 17·6	+48 50	7·1, 7·2	97·2	0·67	4	37·30
Σ 1864 (π Boo)	14 37·2	+16 44	4·9, 6·0	105·7	5·79	3	36·55
Σ 1865 (ζ Boo)	14 37·6	+14 3	4·4, 4·8	131·8	1·10	3	35·62
				132·9	1·06	4	36·31
				133·3	1·14	7	37·30
Σ 1888 (ξ Boo)	14 47·9	+19 24	4·7, 6·6	15·9	4·50	4	35·53
				14·1	4·52	4	36·31
				13·3	4·76	3	37·26
Σ 1909 (44 Boo)	15 1·4	+47 56	5·2, 6·1	247·5	2·55	3	37·42
Σ 1937 (η CrB)	15 20·1	+30 33	5·2, 5·7	250·8	0·48[e]	5	35·60
				291·2	...	4	37·36
Σ 1998 (ξ Sco) AB	16 0·3	−11 11	4·9, 5·2	24·5	1·23	5	37·40
AC	...	...	...	60·4	6·99	5	37·38
BC	...	...	...	55·4	8·15	5	37·38
Σ 2032 (σ CrB)	16 11·9	+34 3	5·0, 6·1	223·6	5·51	3	36·67
Σ 2052	16 25·6	+18 33	7·5, 7·5	235·2	0·66	4	37·39
Σ 2114	16 58·4	+ 8 33	6·6, 7·8	171·8	1·21	3	35·67
OΣ 338	17 48·6	+15 20	7·2, 7·4	5·3	0·79	3	35·67
Σ 2244	17 53·2	+ 0 4	6·9, 7·1	282·4	0·58	4	36·62
Σ 2264 (95 Her)	17 58·3	+21 36	4·9, 4·9	258·6	6·10	3	36·64
Σ 2262 (τ Oph)	17 59·0	− 8 11	5·0, 5·7	266·2	1·96	3	36·61
Σ 2272 (70 Oph)	18 1·7	+ 2 33	4·1, 6·1	118·3	6·71	3	35·64
				117·7	6·65	3	36·60
				117·1	6·57	3	37·71
Σ 2289	18 6·7	+16 27	6·0, 7·1	225·3	1·20	3	36·65
OΣ 358	18 32·5	+16 55	6·8, 7·2	180·8	1·92	2	36·74
Σ 2375	18 41·8	+ 5 25	6·2, 6·6	117·2	2·40	3	35·64
Σ 2382 (ϵ^1 Lyr)	18 41·9	+39 36	5·1, 6·1	5·1	2·88	4	35·70
Σ 2383 (ϵ^2 Lyr)	18 41·9	+39 32	5·1, 5·4	111·6	2·33	4	35·70
Σ 2404	18 47·3	+10 53	5·8, 7·0	182·2	3·44	3	36·61
Σ 2525	19 23·5	+27 10	7·4, 7·6	300·0	1·30	3	35·65
Σ 2579 (δ Cyg)	19 42·6	+44 56	3·0, 7·9	262·8	1·87	3	36·66
				260·0	1·89	4	37·78
Σ 2583 (π Aql)	19 45·2	+11 38	6·0, 6·8	113·0	1·41	5	36·59
Σ 2596	19 50·6	+15 6	7·2, 8·6	322·0	1·79	4	36·61
Σ 2605 (ψ Cyg)	19 53·8	+52 14	5·0, 7·5	178·6	3·12	4	36·70
OΣ 395 (16 Vul)	19 58·8	+24 43	5·8, 6·2	110·5	0·81	5	36·63
Σ 2644	20 8·8	+ 0 39	6·8, 7·1	27·2	2·95	3	36·65
OΣ 413 (λ Cyg)	20 44·5	+36 13	5·0, 6·3	39·6	0·69	4	37·81

Star	1925 R.A.	1925 Dec.	Mags.	Position Angle	Distance	No. of Nights	Date 1900 +
	h m	° ′		°	″		
Σ 2762	21 5·5	+29 54	6·0, 8·0	308·0	3·39	3	36·66
OΣ 437	21 17·7	+32 8	6·9, 7·6	32·6	1·96	2	36·66
Σ 2799	21 25·2	+10 46	6·6, 6·6	284·5	1·53	3	36·67
β 172 (51 Aqr)	22 20·1	− 5 13	5·6, 5·7	161·4	0·58	3	36·69
Σ 2909 (ζ Aqr)	22 25·0	− 0 24	4·4, 4·6	290·7	2·62	4	35·72
OΣ 483 (52 Peg)	22 55·5	+11 20	6·2, 7·7	247·9	0·73	4	36·76
OΣ 494	23 17·1	+21 33	7·4, 8·1	81·2	3·34	2	36·73
Σ 3050 (37 And)	23 55·7	+33 19	6·0, 6·0	242·0	1·94	3	35·74

Notes

a. γ^2 And notched only in 1937; scarcely divided in 1936.
b. Measures of Castor in 1934 accidentally omitted from *M.N.*, **95**, 186.
c. η CrB notched in 1935; only elongated in 1937.